A new methodology for assessment the stability of Ground Motion Prediction Equations

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Abstract Selection of an appropriate Ground Motion Prediction Equation (GMPE) is a key element within the Seismic Hazard Analysis (SHA). A new methodology is introduced in this paper in order to assess the stability of GMPEs. The proposed methodology is named Re-Sampling Analysis (RSA), in which it evaluates the sensitivity of GMPEs under a given subset of re-sampled data. The model bias is calculated, in the proposed methodology, on the basis of the statistical hypothesis tests for different residual components. Four Next Generation Attenuation (NGA) models were evaluated in order to investigate their stability by means of statistical RSA within their own databases. The case study results show that some of the considered GMPEs are quite sensitive to their own databases. Hence, the RSA methodology, as a stability criterion, has been proposed as a practical tool within the GMPE development and also as an effective and complementary tool for selection of the most appropriate GMPE within a SHA.

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Introduction

A reasonable prediction of the expected ground-motion parameters, such as Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV), and Spectral Acceleration (SA), plays a fundamental role in the reliable assessment of seismic hazard. Ground Motion Prediction Equations (GMPEs) are the most important components that significantly affect the Probabilistic Seismic Hazard Analysis (PSHA) results. The growing quantity and quality of ground-motion information on recordings, in different databases, has resulted in numerous regional and worldwide GMPEs through recent decades (Douglas, 2011). However, it has been observed that different global GMPEs can result in quite different outputs for various tectonic regimes (Mousavi et al., 2012; Kaklamanos and Baise, 2011 and Shoja-Taheri et al., 2010). The sources of these differences are the considered database, the mathematical shape of GMPEs, the procedures considered for the development of GMPEs and the chosen input variables. The selection of an appropriate GMPE is one of the primary components of any SHA for a specific seismic region since there are many new emerged GMPEs in the literature (Toro, 2006).

There are many statistical and mathematical methods to assess the compatibility between the observed and predicted data such as: chi-square test, Kolmogorov-Smirnov test, Error comparison tests (e.g. Root Mean Square Error (RMSE) and Mean Absolute Error (MAE)), Coefficient of determination, Nash-Sutcliffe efficiency coefficient, Variance reduction, aspect of bias, and other goodness-of-fit statistics. Furthermore, two different likelihood-based schemes, that are LH method
(Scherbaum et al., 2004) and LLH method (Scherbaum et al., 2009), were emerged in order to evaluate the GMPEs. LH method is a well-designed tool for ranking GMPEs which measures not only the model fitness, but also the primary statistical assumptions (Scherbaum et al., 2004). However, the dependence on the ad hoc assumptions is still a challenge. Therefore, the information-theoretic method has been emerged to overcome the dependence of the results on the ad hoc assumptions e.g. sample size and significant thresholds (Scherbaum et al., 2009). In addition, these two likelihood-based methods, as well as other classical residual analysis methods, inspired the researchers to introduce the Euclidean Distance-Based Ranking (EDR) method by consideration of the ground-motion uncertainty and measuring the bias between the observed data and median estimations of GMPEs (Kale and Akkar, 2013).

There are several comprehensive studies on selection and ranking the GMPEs based on a given set of candidate local and global GMPEs by means of the classical and modern methodologies (Bindi et al., 2006; Scassera et al., 2009; Shoja-Taheri et al., 2010; Kaklamanos and Baise, 2011; Mousavi et al., 2012). It is worth noting that the database was assumed to be fixed during these studies. However, the sensitivity of GMPE on the given Ground Motion Record (GMR) database still needs more investigation. Therefore, this study presents a new methodology for assessment of the stability of GMPEs based on a given database. The authors believe that this feature has been neglected while generating the predictive models. The proposed methodology, named Re-sampling Analysis (RSA), is based on the definition of a hypothesis test in order to estimate the existence of bias for the different types of residual components (i.e. inter-event residuals, intra-event residuals, and total residuals) versus different input parameters such as moment magnitude, source to site distance, and shear-wave velocity. In this paper, in order to show the applicable accomplishment of the RSA method, the Next Generation Attenuation (NGA) models (Power et al., 2008), which were developed in 2008, have been chosen and evaluated via RSA approach.
Summary of NGA GMPEs and their datasets

In 2008, after five-years effort, the Pacific Earthquake Engineering Research (PEER) center’s Next Generation Attenuation (NGA) project released a new series of ground-motion prediction equations through a comprehensive research program for shallow crustal earthquakes in the Western North of America (Power et al., 2006). The NGA metadata information, that has been used to develop the NGA GMPEs, is relatively large (i.e. 3551 recordings from 173 earthquake events) in order to decrease the aleatory variability and also improve the estimation quality in the case of near-source ground-motions. These GMPEs consist of Abrahamson and Silva (2008) (AS08), Boore and Atkinson (2008) (BA08), Campbell and Bozorgnia (2008) (CB08), Chiou and Youngs (2008) (CY08), and Idriss (2008) (I08). The NGA GMPEs are summarized in Table 1 including the validity range of the magnitude, distance measure, and shear-wave velocity. Note that I08 model only includes rock site (assumed to be sites with $V_{S30} \geq 450$ m/s) in which this significant difference isolates the Idriss model from the other models. Therefore, this model is excluded for further investigation in this paper.

Table 1. Summary of the NGA GMPEs, indicating distance metric and conditions of use.

As illustrated in Table 2, there is a set of comprehensive and different types of components to be employed within the NGA GMPEs (Kaklamanos et al., 2010). Accordingly, in this study, the result of Kaklamanos’s technical note has been used to determine unknown parameters of the NGA flatfile in order to reduce the uncertainties and convert all input variables of GMPEs into a unique definition (Kaklamanos et al., 2010 and 2011).
Table 2. Explanatory variables for implementation within the NGA GMPEs (Kaklamanos et al., 2010).

The NGA GMPEs are known as global predictive models and the only constraint is that the study region should be tectonically active with shallow crustal earthquakes. Therefore, numbers of quantitative comparisons have been employed in order to examine GMPEs on the basis of different seismic regions and using recorded or synthetic data sets e.g. Ghasemi et al. (2009), Shoja-Taheri et al. (2010), and Mousavi et al. (2012) which evaluated the NGA GMPEs in the case of Iran seismic plateau database. Campbell and Bozorgnia (2006), Stafford et al. (2008), and Scasserra et al. (2009) compared the NGA GMPEs with local models for European database, and Graves et al. (2008), Olsen et al. (2008, 2009), Star et al. (2008, 2010, 2011), and Kaklamanos and Baise (2011) examined the NGA models for different databases in the California.

In this study, the NGA GMPEs have been evaluated by means of the RSA method in order to assess their stability. Each GMPE was investigated based on its own database, which was implemented in the stage of GMPE development. According to this point, CB08, AS08, BA08, and CY08 models were examined, respectively, based on 1561 recordings from 64 earthquakes, 2754 recordings from 135 earthquakes, 1574 recordings from 58 earthquakes, and 1950 recordings from 125 earthquakes. Figure 1 shows the distribution of moment magnitude versus distance metric measures e.g. the closest distance to the rupture plane (R_{RUP}) and the horizontal distance to the surface projection of the rupture (R_{RB}) for the four NGA databases.

Figure 1. Distribution of recordings with respect to the moment magnitude (M_w) and distance metric measures (R_{RUP}, R_{RB}) for the database which has been used in each NGA model, (a) CB08, (b) AS08, (c) BA08, and (d) CY08.
Each GMPE is obviously obtained based on a specific ground-motion database. Any further earthquake event can update this database in future. However, a small change in the chosen ground-motion database should not significantly affect on the GMPE’s outputs. In other words, if a ground-motion estimator is strongly sensitive to a small change in the ground-motion database, then, the predicted values may not be so reliable and the aleatory variability is remarkable in this case. This issue is demonstrated in Figure 2 for two different random subsets of CB08 database with 1000 GMRs (CB08 database consists of 1561 records). As it is shown in Figure 2, the bias is completely different for these two selected subsets within the total database. Hence, the selected GMPE can predict quite acceptable or unacceptable results under different subsets of a general database with a constant number of records.

Figure 2. The comparison of the bias for two different subsets with the same number of GMRs in the case of CB08 model (T=0.0). (a) No biased, (b) biased. (The solid line is the fit line of the intra-event residuals versus $R_{RUP}$ by linear regression)

To quantify this phenomenon, the authors have proposed the Re-Sampling Analysis (RSA) methodology to assess the sensitivity of the GMPEs to the selected datasets. The basic idea of the proposed methodology is to quantify the induced bias of the residuals versus different types of seismic input parameters. In order to denote the amount of bias and interpretation of the outcomes, a hypothesis test was defined to generate statistical p-values. The null hypothesis was defined as an unbiased fitted line at the predetermined 0.05 significance level which was generated by linear regression with respect to the given data. Additionally, the p-value, in the hypothesis test, which is used in the proposed methodology, is defined as the probability of obtaining a value of the test
statistic as extreme (or more extreme) than the value computed from the sample. The main steps of the methodology are given in Figure 3, and are summarized as the following steps:

1) Select a GMPE.

2) For each GMPE, a reduced number of GMRs, let say N, is selected based on uniformly random number selection with respect to moment magnitude and distance measure distribution (N ≤ maximum number of GMRs).

3) The p-values corresponding to the residual components (i.e. inter-event residual, intra-event residual) versus different types of seismic input parameters (e.g. moment magnitude, rupture distance, Joyner-Boore distance, and shear-wave velocity) and also the amount of different types of statistical indices (e.g. LLH, R-squared, RMSE, MAE and any other potential statistical goodness of fit), are calculated based on the reduced database which was defined in Step 2.

4) Steps 2 and 3 are repeated for K times to avoid any bias from the random selection process (e.g. K=100).

5) Steps 2, 3 and 4 are repeated from an initial assumption number of subsets of the data (let say N = 100) to the maximum number of the considered entire data (GMRs or events) by a constant increment (let say 100, in this study).

6) The obtained indicators that were calculated in Step 3 are shown versus N (see Figure 4). Additionally, the median of each indicator, in Step 3 for N samples is calculated as a final indicator. It makes possible (and more sense) to show the final indicator in one plot for all GMPEs.

**Figure 3.** The main steps in the proposed RSA methodology.
Implementation of RSA on NGA GMPEs

In order to show the applicability of the RSA method, four NGA GMPEs have been selected with their referenced metadata. In this study, different types of residuals for NGA GMPEs are defined by means of the general random effects as written in Eq.(1):

\[ LnY_{ij} = Ln\tilde{Y}_{ij} + \eta_i + \varepsilon_{ij} \]  \hspace{1cm} (1)

where \( \eta_i \) is the random effect (also known as the inter-event residuals) for the \( i^{th} \) earthquake, and \( Ln\tilde{Y}_{ij} \), \( LnY_{ij} \), and \( \varepsilon_{ij} \) are, respectively, the median estimate, the observed value, and the intra-event variation of the \( j^{th} \) recording for the \( i^{th} \) earthquake. \( \eta_i \) and \( \varepsilon_{ij} \) are assumed to be independent and both are normally distributed, respectively, with variances \( \tau^2 \) and \( \sigma^2 \). In this case, it is positive to relate \( \eta_i \) and \( \varepsilon_{ij} \) to the total model residual, that is defined as the difference between the observed and predicted values. Hence, the total model residual is calculated as written in Eq.(2):

\[ r_{ij}^{[total]} = LnY_{ij} - Ln\tilde{Y}_{ij} = \eta_i + \varepsilon_{ij} \]  \hspace{1cm} (2)

The inter-event and intra-event residuals in Eq.(2) are defined by Eq.(3) and Eq.(4) (Abrahamson and Youngs, 1992):

\[ r_{i}^{[inter]} = \eta_i = \frac{\tau^2 \sum_{j=1}^{N_i} r_{ij}}{\tau^2 N_i + \sigma^2} \]  \hspace{1cm} (3)

\[ r_{ij}^{[intra]} = \varepsilon_{ij} = r_{ij} - \eta_i \]  \hspace{1cm} (4)
It should be noted that if an earthquake has just a single record, then the percentage of the residuals, that is assigned to the inter-event term, is given by the ratio $\frac{\tau^2}{\tau^2 N_i + \sigma^2}$. On the contrary, if there are a large number of recordings from an event, then the inter-event term becomes the mean residuals for that event (Abrahamson and Youngs, 1992).

By given the mentioned points, Figure 4 shows the RSA results for a constant period ($T = 0.01$ s) in the case study of CB08 GMPE for 100 uniformly random selected databases ($K = 100$), available in the electronic supplement to this article. As seen in Figure 4, the stability of the candidate model is shown versus source parameter (e.g. $M_w$), path parameters (e.g. $R_{RUP}$, $R_{JB}$), and site parameter (e.g. $V_{S30}$) by means of the inter-event (Eq.(3)) and intra-event (Eq.(4)) residuals. Also, some of the modern and traditional statistical tests (e.g. LLH index, $R^2$-squared index, RMSE and MAE indices) are implemented as error terms with attention to the total residuals (see Eq.(2)). As CB08 model has been obtained based on the 1561 records from 64 events, however, there is not enough consistency for subsets of the whole database even the total number of data (GMRs and events) is reached. It is worth to mention that an unbiased model should represent an ascending performance while the sample size is increased. In other words, as the subset gets more data, the less bias should be observed. As seen in Figure 4, the RSA results show stable trends for inter-event residuals versus moment magnitude and intra-event residuals versus shear-wave velocity, rupture distance, and Joyner-Boore distance. The RSA results for LLH, $R^2$, and error terms criteria (RMSE and MAE) are also shown in Figure 4e to Figure 4h in which they represent good convergence when the total number of database is reached. The median value (diamond point) in each RSA case is also shown in Figure 4 to express the trend of RSA versus different numbers of GMRs in the subset.
**Figure 4.** CB08 RSA for 100 uniformly random selected databases. (a) Inter-event residuals versus $M_w$, (b) Intra-event residuals versus $V_{S30}$, (c) Intra-event residuals versus $R_{RUP}$, (d) Intra-event residuals versus $R_{JB}$, (e) RMSE, (f) MAE, (g) LLH, (h) $R^2$. (The diamond points show the median of p-values).

The RSA methodology was also applied for three other NGA GMPEs. In order to have accurate interpretation of the sensitivity of the GMPEs, based on the RSA approach, the median p-values was calculated for different sorts of residuals versus different earthquake parameters and also for different statistical indices.

**Visual comparison of RSA results for NGA GMPEs**

To increase the reliability of the obtained RSA results, the K factor in step 3 in the RSA methodology should be defined appropriately. The process of choosing the optimized K factor is summarized as the following steps:

1) Select an initial assumption for the number of subsets (GMRs or events), let say $K=50$, with a constant number of GMRs, let say $N=1000$, in this study.

2) The p-values corresponding to the residuals, versus different types of seismic input parameters, are calculated based on the chosen subsets, which was defined in Step 1 (e.g. intra-event residuals versus $R_{RUP}$).

3) The median p-values is calculated and stored.

4) Steps 1, 2 and 3 are repeated for $T$ times to avoid any bias from the random selection process, let say $T=50$, in this study.

5) The interval between the maximum and minimum of the stored median p-values in step 3 is calculated.
6) Steps 1 to 5 are repeated by a constant increment (e.g. 50) in K parameter until the interval in step 5 is less than 5%.

7) The obtained intervals, which were calculated in Step 5, are shown versus K factor. Figure 5 shows the procedure of obtaining the optimized K factor according to the aforementioned steps and Figure 6 shows the results of the optimized K factor in this study. As seen in Figure 6, the median p-values are obtained for CB08 model in the case of PGA based on intra-event residuals versus R_{RUP} for the data subsets with 1000 GMRs with 50 iterations. The optimized K factor is equal to 400 in this case. Therefore, the rest analyses in this paper are provided by implementing this K factor.

Figure 5. Flowchart of obtaining the optimized K factor for RSA method in this study.

Figure 6. The optimized number of random databases for RSA method.

In order to prove the stability of the candidate GMPE models, 400 uniformly distributed databases (with the optimized K = 400) were implemented. The median of RSA results was calculated for NGA GMPEs and the results based on the inter-event and intra-event residuals and also the selected statistical and mathematical tests are shown in Figure 7. As an interesting fact, the results based on the primary statistical tests in Figure 7 (e.g. the LLH criterion as an information-theoretic model selection method, error terms (RMSE and MAE), and coefficient of determination index) are completely independent of the sample size and follow a constant trend. The source of this fact is that all of those approaches employ a kind of averaging procedure within their methodologies. As a fact, as seen in Figure 4, RMSE, LLH, MAE and R-squared variations decrease by increasing the sample size (N). On the other hand, the median RSA results do not show a
constant trend toward different seismic input parameters versus inter-event and intra-event residuals for NGA GMPEs. This phenomenon allows us to focus on the RSA results in order to compare the GMPE models.

Figure 7 shows the median of the RSA results of the NGA GMPEs by means of inter-event and intra-event residuals in order to assess the direct effect of different parameters on the RSA process. As seen in Figure 7, all NGA GMPEs show approximately consistent performance with ascending trend toward inter-event residuals versus moment magnitude. As a result, AS08 has slightly descending performance towards shear-wave velocity and distance measures with respect to intra-event residuals. On the other hand, BA08 model has ascending trends towards source parameter (M_w) that shows good consistency of the predicted model; however, it has descending trends towards the Joyner-Boore distance, closest distance to the rupture plane and shear-wave velocity. It means that BA08 model shows the existence of bias with respect to the site parameter (V_{S30}) as well as the path parameters (R_{RUP} and R_{JB}) for the case study of PGA. As a conclusion, CB08 and CY08 models demonstrate more stable performance toward its referenced database than the other candidate NGA GMPEs for this case study of PGA. This conclusion can be endorsed by the other goodness-of-fit statistics and statistical tests which were applied in this study e.g. information-theoretic method (LLH), error terms (RMSE and MAE), and coefficient of determination as seen in Figure 7(e) to 7(h). As illustrated in Figure 7, CB08 has the lowest LLH, RMSE, and MAE values and the highest R^2 value.

As a crucial point, the comparison of BA08 model’s statistical results with the RSA results indicates the necessity of the RSA scheme through the GMPEs selection process. It should be emphasised that this conclusion is only valid in the case study of PGA and we cannot broaden it through other periods in this stage of research. It is worth to mention that the RSA method can be a complementary approach for selecting the appropriate GMPE models.
Figure 7. NGA GMPEs median of RSA for 400 uniformly random selected databases. inter-event residual towards (a) $M_w$, intra-event residuals towards (b) $V_{S30}$, (c) $R_{RUP}$, (d) $R_{JBL}$, statistical indices (e) RMSE, (f) MAE, (g) LLH, (h) $R^2$.

Conclusion

A new assessment methodology, for evaluating and selecting the GMPEs, has been introduced in the current paper. The proposed procedure is based on re-sampling of uniformly random selected data subsets within a general database in order to determine the bias. The Re-Sampling Analysis method, named RSA method, can be compiled for different kinds of earthquake parameters and also different goodness-of-fit statistics or statistical tests. The authors believe that this method is a robust strategy in order to test the sensitivity of the predictive models as a pivotal feature for pre-selecting GMPEs in PSHA.

In this study, in order to show the general aspects of the proposed procedure, four NGA GMPEs as worldwide predictive models, were selected and the RSA method was applied by considering their own databases for a specific period ($T = 0.0 \text{ s}$). In some cases, the results of this study represent instability and unavoidable bias of the residuals versus the moment magnitude, distance measures, and shear-wave velocity. Also the RSA results indicate the independence of the information-theoretic method (LLH), coefficient of determination ($R^2$), and error terms (RMSE and MAE) to the sample size. The RSA method can be a useful tool to improve the ability of selecting the most appropriate GMPE visually by means of model bias trend.

The RSA methodology can be used as an essential and complementary testing method. On the other hand it can be one of the most potent selecting tool of GMPEs through PSHA in specific sites and also, can be a beneficial tool for the development of GMPEs. It is worth emphasizing that more
studies are needed for the purpose of applying the proposed method in the case of weighting GMPEs within a logic-tree process.

Data and Resource

To implement NGA GMPEs in the current study, essential information about NGA project, as well as NGA Flatfile used for development of NGA models and numerical programs have been employed by means of publicly available resource on the Pacific Earthquake Engineering Research Next Generation Attenuation Project web site at: http://peer.berkeley.edu/ngawest/index.html (last accessed August 2013).

References


Campbell, K. W., and Y. Bozorgnia (2008). NGA ground-motion model for the geometric mean horizontal component of PGA, PGV, PGD and 5% damped linear elastic response spectra for periods ranging from 0.01 to 10 s, *Earthq. Spectra* **24**, 139-171.


Figure 1. Distribution of recordings with respect to the moment magnitude ($M_w$) and distance metric measures ($R_{rup}$, $R_{jb}$) for the database which has been used in each NGA model, (a) CB08, (b) AS08, (c) BA08, and (d) CY08.
Figure 2. The comparison of the bias for two different subsets with the same number of GMRs in the case of CB08 model (T=0.0). (a) No biased, (b) biased. (The solid line is the fit line of the intra-event residuals versus Rrup by linear regression.)
Figure 3. The main steps in the proposed RSA methodology.
Figure 4. CB08 RSA for 100 uniformly random selected databases. (a) Inter-event residuals versus $M_w$, (b) Intra-event residuals versus $V_{S30}$, (c) Intra-event residuals versus $R_{rup}$, (d) Intra-event residuals versus $R_{jb}$, (e) RMSE, (f) MAE, (g) LLH, (h) $R^2$. (The diamond points show the median of p-values).
Figure 5. Obtaining the optimized number of random databases for RSA method.
Figure 6. Obtaining the optimized K factor for RSA method in this study.
Figure 7. NGA GMPEs median of RSA for 400 uniformly random selected databases. Inter-event residual towards (a) $M_w$, intra-event residuals towards (b) $V_{S30}$, (c) $R_{\text{RUP}}$, (d) $R_{\text{JB}}$, statistical indices (e) RMSE, (f) MAE, (g) LLH, (h) $R^2$. 

M$_w$, intra-event residuals towards (b) $V_{S30}$, (c) $R_{\text{RUP}}$, (d) $R_{\text{JB}}$, statistical indices (e) RMSE, (f) MAE, (g) LLH, (h) $R^2$. 

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Table 1

Summary of the NGA GMPEs, indicating distance metric and conditions of use.

<table>
<thead>
<tr>
<th>GMPE</th>
<th>Abbreviation</th>
<th>Dominant Region</th>
<th>Distance metric</th>
<th>(^* M_w)</th>
<th>Distance (km)</th>
<th>(^{\dagger}V_{S30})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campbell and Bozorgnia 2008</td>
<td>CB08</td>
<td>Western US and California</td>
<td>(^{\dagger}R_{RUP})</td>
<td>4.0-7.0/8.0/8.5</td>
<td>0-200</td>
<td>150-1500</td>
</tr>
<tr>
<td>Boore and Atkinson 2008</td>
<td>BA08</td>
<td>Western US and California</td>
<td>(^{\dagger}R_{JB})</td>
<td>5.0-8.0</td>
<td>0-200</td>
<td>180-1300</td>
</tr>
<tr>
<td>Chiou and Youngs 2008</td>
<td>CY08</td>
<td>Western US and California</td>
<td>(R_{RUP})</td>
<td>4.0-8.0/8.5</td>
<td>0-200</td>
<td>150-1500</td>
</tr>
<tr>
<td>Abrahamson and Silva 2008</td>
<td>AS08</td>
<td>Western US and California</td>
<td>(R_{RUP})</td>
<td>5.0-8.5</td>
<td>0-200</td>
<td>180-1500</td>
</tr>
<tr>
<td>Idriss 2008</td>
<td>I08</td>
<td>Western US and California</td>
<td>(R_{RUP})</td>
<td>5.0-8.0/8.5</td>
<td>0-200</td>
<td>450-900</td>
</tr>
</tbody>
</table>

\(^{\dagger}M_w\), Moment Magnitude (depending on fault mechanism)

\(^{\dagger}V_{S30}\), Shear-wave velocity (m/sec)

\(^{\dagger}R_{RUP}\), Closest distance to the rupture plane (Rupture distance)

\(^{\dagger}R_{JB}\), Horizontal distance to the surface projection of the rupture (Joyner-Boore distance)
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Source Parameters</th>
<th>Path Parameters</th>
<th>Site Parameters</th>
<th>NGA GMPEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment magnitude, $M_w$</td>
<td></td>
<td></td>
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<td>AS08</td>
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<tr>
<td>Depth to top of rupture, $Z_{TOR}$</td>
<td></td>
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<td>BA08</td>
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<td>Down-dip rupture width, $W$</td>
<td></td>
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<td>CB08</td>
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<td>Fault dip, $\delta$</td>
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<td>CY08</td>
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<tr>
<td>Style-of-faulting flag (function of rake angle, $\lambda$)</td>
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<td></td>
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<tr>
<td>Aftershock flag</td>
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<tr>
<td>Closest distance to the rupture plane, $R_{RUP}$</td>
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<td>AS08</td>
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<tr>
<td>Horizontal distance to the surface projection of the rupture, $R_{JB}$</td>
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<td>BA08</td>
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<tr>
<td>Horizontal distance to top edge of rupture measured perpendicular to the strike, $R_x$</td>
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<td>CB08</td>
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<tr>
<td>Hanging-wall flag</td>
<td></td>
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<td>CY08</td>
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<tr>
<td>Time-averaged shear wave velocity over the top 30 m of subsurface, $V_{S30}$</td>
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<td>Depth to bedrock or specific shear wave velocity horizon ($Z_{1.0}$, $Z_{2.5}$)</td>
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<td>PGA (or SA) on rock, as baseline for nonlinear site response</td>
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</table>

Explanatory Variables for Implementation within the NGA GMPEs. (Kaklamanos \textit{et al.}, 2010).
Figure

(a) Subject No. 1
- number of GMRE: 1000
- p-value: 0.988

(b) Subject No. 2
- number of GMRE: 1080
- p-value: 0.145

Intraclass Residuals vs. $R_{rup}$ (km)
Start

(1) Select a GMPE

K_{max}: A constant value for repeating the process (e.g. 100).
K=1

(2) Select a uniform random subset of available data set with N records.
(N \leq \text{Max number of GMRs})

(3) Calculate p-values
(e.g. M vs. inter-event residuals, R and VS30 vs. intra-event residuals, or different statistical indexes vs. total residuals)

(4) Increase K by one

Yes

K \leq K_{max} \rightarrow Yes

No

(5) Increase N by a constant increment
(e.g. 100)

Yes

N \leq \text{max. number of GMR in database} \rightarrow Yes

No

(6) Show median p-values and different statistical indexes [step 3] for a specific N.

End
The interval between maximum and minimum of 50 median of p-values by intra-event residuals vs. Rrup for 150 different databases with 1000 GMRs.

The optimized K factor.