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Estimating the annual probability of failure using improved progressive incremental dynamic analysis of structural systems

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SUMMARY

A methodology based on the progressive incremental dynamic analysis has been introduced in this paper to estimate the structural response and the corresponding annual probability of failure. The proposed methodology employs the genetic algorithm optimisation technique and an equivalent single-degree-of-freedom system corresponding to the first-mode period of a considered structure. The proposed methodology can significantly reduce the number of ground motion records needed for estimating the annual probability of failure. The numerical results indicate that the proposed method can effectively reduce the computational effort needed for computation of probability of failure for the first-mode dominated structures, which is advantageous as the structure becomes larger. A relatively huge set of single-degree-of-freedom systems as well as three multi-degree-of-freedom systems including 3, 8 and 12 storeyed reinforced concrete structures was taken into account to test the proposed methodology. It has been shown that the probability of failure can be estimated within ±15% error with 95% confidence. The proposed method can speed up the decision-making process in the probability-based seismic performance assessment of structures, and it also incorporates the randomness of strong ground motions explicitly. Copyright © 2012 John Wiley & Sons, Ltd.

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KEYWORDS: progressive IDA; fragility curve; genetic algorithm; probability of failure

1. INTRODUCTION

Structural analysis often involves large uncertainties, especially when the input is highly uncertain, as is the case of seismic loading. The probability-based methods attempt to deal with this uncertainty in the seismic design and assessment of structures. The performance evaluation of structures is often described in terms of demand and capacity, where the demand can be any structural response of interest (shear, moment, drift, etc.) and the capacity is the maximum structural response in which the structural behaviour is acceptable. The seismic demand and capacity and their distributions can be calculated by means of incremental dynamic analysis (IDA), which is commonly used for different nonlinear analysis applications (Vamvatsikos and Cornell, 2002; Liao *et al*., 2007; Tagawa *et al*., 2008). IDA employs several response-history analyses for a given ground motion record by increasing the intensity measure (IM) until the collapse occurs. This process is repeated for a sufficient number of ground motion records to determine the median collapse capacity and the record-to-record variability. Determining the potential of collapse in structures, because of its importance in decision-making and performance-based earthquake engineering, has received much attention, and there have been several studies on the subject (Roeder *et al*., 1993a, 1993b; Challa and Hall, 1994; Krawinkler *et al*., 2006;

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Goulet *et al*., 2007; Zareian and Krawinkler, 2007; Liel *et al*., 2009; Han *et al*., 2010). A comprehensive review of some analytical methods can be found in a state of the art article by Villaverde (2007).

One of the most well-known methodologies for the probability assessment of structures, which was developed for the SAC2000 project (Cornell *et al*., 2002), involves three random elements: the ground motion intensity, the displacement demand and the displacement capacity. The ground motion intensity is selected as the spectral acceleration at the first period of a given structure ($S_a(T_1)$) for a specific damping. The combination of the first two elements produces the drift hazard curve, $H_D(d)$, and combining this curve with the third element determines the annual probability of the performance level not being met, P_{PL}. Furthermore, as an alternative, P_{PL} can be computed directly using IM-based approach (Jalayer and Cornell, 2003). If the performance level is set to be the collapse capacity, the P_{PL} would become the 'annual probability of failure'. The SAC2000 methodology provides a closed form solution for determining these values, but there are some shortcomings in the closed form solution rooted in the simplifying assumptions, e.g. a fixed value for dispersion, structural type limits and so on, but these can be avoided by means of the direct IDA analysis.

One of the most challenging issues in IDA is the significant computational effort, which is needed for the nonlinear response-history analyses. This issue even gets more complicated as the structure grows taller in terms of extensive computational effort. To reduce this effort required in IDA calculation, different approximate methods have been introduced, which can be summarized in seven categories. (a) Vamvatsikos and Cornell (2005, 2006) presented SPO2IDA to reduce the required time to obtain IDA curves; (b) Baker and Cornell (2005) introduced the epsilon-based filtration approach to select the ground motion records, which employs the epsilon advantages for reducing the number of ground motion records; (c) Dolšek and Fajfar (2005) showed that the N2 method can also be used for the determination of approximate summarized IDA curves; (d) Han and Chopra (2006) proposed the approximate IDA using modal pushover analysis of the multi-degree-of-freedom (MDOF) system and nonlinear dynamic analysis of corresponding single-degree-of-freedom (SDOF) systems, which can consider higher mode effects but may not be reliable in estimating IDA curves in the case of irregular structures (Vejdani-Noghreiyan and Shooshtari, 2008); (e) Ghafory-Ashtiany *et al*. (2010) tried to classify ground motion records for different structural groups by incorporating the multivariate statistical analysis with the principal component analysis. They classified a wide range of SDOF systems into six different groups and have proposed eight ground motion records for each group to reliably estimate the mean structural response; (f) Mousavi *et al*. (2011) proposed the eta-based filtration approach, which is a more robust approach in comparison with the former epsilon-based filtration approach. (g) Azarbakht and Dolšek (2007, 2011) introduced the progressive IDA (PIDA), which involves a precedence list of strong ground motion records (SGMRs) and is capable of reducing the computational efforts needed to obtain the summarized IDA curves (16th, 50th and 84th fractiles) with reasonable approximation for MDOF systems. The proposed methodology takes advantage of the analysis of a first-mode equivalent SDOF system and optimisation concept using the genetic algorithm (GA). The proposed method is obviously limited to the first-mode dominated structures in its current form.

In this research, an attempt has been made to modify the PIDA optimisation method to estimate P_{PL}. The proposed method was applied to MDOF structures for a given hazard condition to estimate the annual probability of failure. The results are described in Section 4.

2. METHODOLOGY

The objective of the proposed methodology is to reduce the number of required SGMRs for computing the 'annual probability of failure' (P_{PL}) within an acceptable accuracy. The maximum interstorey drift ratio (MIDR) was selected as the engineering demand parameter (EDP). The capacity (or the ultimate limit state), which is the acceptable structural behaviour limit (here selected as the global dynamic instability), should also be represented on the same basis as the demand parameter, MIDR, to make the comparison possible. This methodology uses the PIDA concept, for which a detailed step by step procedure can be found in Azarbakht and Dolšek (2011).

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Probability of failure in IM-based approach, P_{PL}, can be computed as

$$P_{\mathrm{PL}} = \int P\left[S_{\mathrm{a,C}} \leq x\right].\left|\mathrm{d}H_{S_{\mathrm{a}}}(x)\right| = \int F(s_{\mathrm{a}}).\left|\mathrm{d}H_{S_{\mathrm{a}}}(x)\right| \tag{1}$$

where $F(s_{\mathrm{a}})$ is the fragility function at spectral acceleration (s_{a}) and $\mathrm{d}H_{S_{\mathrm{a}}}(x)$ is the differential of the seismic hazard curve. The lognormal cumulative distribution function (CDF) is often employed to model system and component fragility (Shinozuka *et al.*, 2000) and, besides convenience, has some theoretical justifications (Ellingwood, 1990). Different studies on steel and concrete frames have shown that the lognormal CDF provides a good fragility model in the inelastic range of response (Hwang and Jaw, 1990; Singhal and Kiremedjian, 1996; Dymiotis *et al.*, 1999; Song and Ellingwood, 1999). The multiplication of failure fragility curve and hazard derivative is referred to, herein, as the 'hazard derivative-fragility product'.

The original Error function introduced by Azarbakht and Dolšek (2007, 2011) is shown in Equation (2). In this equation, s is the number of selected ground motion subsets to estimate the fractiles, which is a factor of three as three fractiles are to be estimated, EDP is the engineering demand parameter of the simple model, IM is the intensity measure for the IDA, and $\Delta IM(s, f)$ is the difference in the IM corresponding to the 'original' and 'estimated' fth summarized IDA curves. The 'or' as in $IM_{\mathrm{or}}(f)$ refers to original values, and f refers to the fth summarized IDA curve of interest (16%, 50%, etc.). $EDP_{\max}(s, f)$ is the maximum of the engineering demand parameters corresponding to the global dynamic instability of the 'approximate' or 'original' fth summarized IDA curve, and $EDP_{\max,\mathrm{or}}(f)$ is the engineering demand parameter corresponding to the capacity point of the 'original' fth summarized IDA curve. The parameters $\Delta IM(s, f)$ and $EDP_{\max}(\mathrm{s}, f)$ depend on the s selected subsets of the ground motion records, which were used in determining the 'approximate' fth summarized IDA curve.

$$Error_{\mathrm{or}}(s,f) = 100\left[\frac{\int_0^{EDP_{\max}(s,f)} |\Delta IM(s,f)|\mathrm{d}(EDP)}{\int_0^{EDP_{\max,\mathrm{or}}(f)} IM_{\mathrm{or}}(f)\mathrm{d}(EDP)}\right] \tag{2}$$

But given Equation (1), a better estimation of fragility for an assumed hazard could lead to a better approximation of P_{PL}. Thus, besides other parameters necessary in the PIDA method, a lognormal mean value and dispersion should also be taken into account by assuming a lognormal distribution for the collapse capacity. As a whole, selecting the error function for GA is a matter of trial and error considering the physics of the problem to be handled. This additional constraint can be effectively included in the original fitness function by including some additional terms as shown in Equation (3), which hereafter is referred to as the improved error function:

$$Error_I(s,f) = 100\left[\frac{\int_0^{EDP_{\max}(s,f)} |\Delta IM(s,f)|\mathrm{d}(EDP)}{\int_0^{EDP_{\max,\mathrm{or}}(f)} IM_{\mathrm{or}}(f)\mathrm{d}(EDP)} + \frac{\left|\mu_{\mathrm{Ln}(IM)_{or}} - \mu_{\mathrm{Ln}\,IM}(s)\right|}{\mu_{\mathrm{Ln}(IM)_{or}}} + \frac{\left|\beta_{IM_{\mathrm{or}}} - \beta_{IM}(s)\right|}{\beta_{IM_{\mathrm{or}}}}\right] \tag{3}$$

Here, $\mu_{\mathrm{Ln}(IM)_{or}}$ is the 'original' logarithmic mean value of the collapse capacity, $\mu_{\mathrm{Ln}(IM)}(s)$ is the 'estimated' logarithmic mean value of the collapse capacity based on selected SGMRs and β is logarithmic standard deviation considering a lognormal distribution of the collapse capacity. The final improved fitness function (Z) that was used in the GA can be defined as

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$$Z = \frac{1}{m}\sum_{s=1}^{m}\sum_{f=1}^{3} Error_I(s,f) \tag{4}$$

To determine the range of error and the minimum number of SGMRs needed for proper estimation of P_{PL}, a set of SDOF systems was analysed in this article. Figure 1 summarizes the steps involved in the determination of the adequacy of the proposed method, which is investigated numerically in Section 3. The steps can be described as follows:

1) Consider an input ground motion scenario of interest.
2) Consider a set of SDOF systems with different periods ranging from 0.1 s to 2 s with different ductility, damping and strain-hardening ratios to cover a range of systems and analyse them for the input SGMRs which was defined in Step 1.
3) Extract the precedence list of SGMRs for each SDOF system by using PIDA with application of the original or the improved fitness function by using Equations (2), (3) and (4).
4) Compute the 'best-estimate' of P_{PL} for a full set of SGMRs for each SDOF system and the 'approximate' P_{PL} for the desired number of SGMRs using the pre-determined SGMRs precedence list and for the selected scenario in the site, which may be obtained from the hazard disaggregation or a standard seismic hazard analysis. The difference of the two P_{PL} values (best-estimate and approximate) divided by the 'best-estimate' value is referred to as *Error*. This error has been computed using different numbers of SGMRs, from six to 24 SGMRs, as they appear in the precedence list. Upon combining numbers of SDOF systems and the different structural or hazard assumptions for each number of SGMRs considered, there are 14,580 such error values.
5) Perform the statistical analysis on the obtained results for each group (number of SGMRs), and decide on the minimum number of required SGMRs, which should be used to obtain an acceptable estimation for P_{PL}.

The analysis of variances (ANOVA), which can compare the central tendencies of the different groups of observations, was used as the statistical approach to determine the minimum number of required SGMRs (Girden, 1991). ANOVA has some restrictive conditions, and violating them could result in unreliable outcomes. The normality of each group and independence of the compared groups are two important conditions of ANOVA. The normality condition is met, but the groups (different number of reduced SGMRs extracted from precedence lists) are not independent in this study. For the case of dependent groups (e.g. different levels of one independent variable, which is the case here),

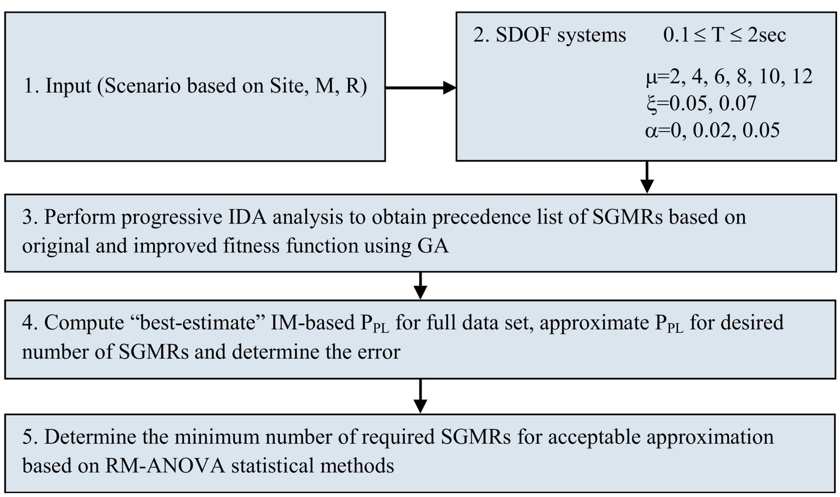

Figure 1. Process of the improved progressive IDA for the purpose of P_{PL} computation.

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another method known as repeated measures ANOVA was chosen to be used (Davis, 2002). It is an extension of the correlated-groups *t*-test, where the main advantage is controlling the disturbance variables or individual differences that could influence the dependent variable. A traditional approach to the analysis of repeated measurements is to (a) perform a standard ANOVA, as if the observations were independent, and (b) determine whether additional assumptions or modifications are required to make the analysis valid (Davis, 2002).

Finally, by the statistical tests for the amount of the error in P_{PL} computation, it is proposed that six ground motion records out of the pre-determined precedence list can be employed for an appropriate estimation of the P_{PL}. The proposed methodology was applied to three MDOF structures in Section 4. The results show that P_{PL} can be estimated within an acceptable error by using only the first six ground motion records in the precedence list obtained using GA and the improved fitness function.

3. ANNUAL PROBABILITY OF FAILURE (P_{PL}) ESTIMATION BASED ON IMPROVED PIDA FOR SDOF SYSTEM SET

To study the efficiency of the proposed methodology and provide a basis for MDOF application, IDA analysis using the Hunt and Fill method (Vamvatsikos and Cornell, 2002) for the considered SGMRs database was performed on a set of SDOF systems. Basic assumptions and system properties as well as the obtained results, their comparison and the statistical analysis, following the steps of Figure 1, are presented in this section. It is worth noting that the computational effort, which has been discussed in this section, is not necessary to be repeated for a real (MDOF) structure as described in Section 4. However, all discussions here, for SDOF systems, are carried out to make a basis for the next section.

3.1. Seismic hazard function and strong ground motion records

In the first step, a simple source, which is capable of producing only a specific magnitude at a specific distance, was considered. For the purpose of sensitivity analysis, different M_w and $R_{rupture}$ values were assumed, which are summarized along with a sample hazard curve as shown in Figure 2. The Campbell and Bozorgnia 2008 (CB 08) (2008) attenuation relationship has been used to determine s_a. Considering s_a, $\lambda_{S_a}(s_a)$ can be computed as $\lambda(S_a > s_a) = vP[S_a > s_a|M, R_{rupture}]$ for a desired return period (TR) and $v = 1/TR$.

The hazard curve for the considered IM can be obtained by repeating the calculation for different s_a values. This simple model suggests the probable use of disaggregation of the seismic hazard analysis and was considered only for simplicity and applicability of the sensitivity analysis of the results.

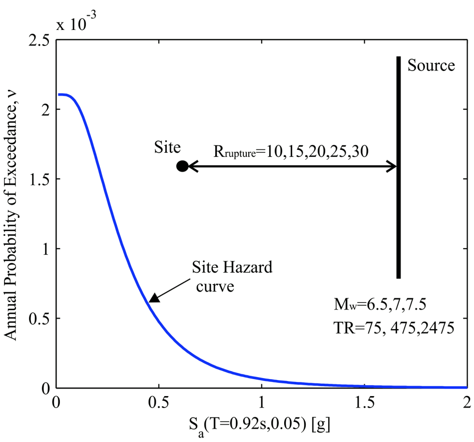

Figure 2. Different parameters used in sensitivity analysis and sample hazard curve for $T = 0.92$ s, $M_w = 7$ and $TR = 475$ years.

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Also, a general far-field ground motion set (FEMA P695, 2009), consisting of 22 ground motion pairs recorded at sites located more than 10 km from the fault rupture, was selected from PEER (2005) to calculate IDA. Figure 3 shows the acceleration spectra of the selected ground motion records, their mean and the respective mean ± standard deviation. The SGMRs details are listed in Table 1.

3.2. SDOF systems properties

As the second step, a set of SDOF systems consisting of 27 periods ranging from $T = 0.1$ to 2 s (from $T = 0.1$ to 1 with 0.05 increments and 1.15, 1.25, 1.35, 1.5, 1.65, 1.75, 1.85 and 2 s), six ductility ratios ($\mu = 2$, 4, 6, 8, 10, 12), two damping ratios ($\xi = 5\%$, 7%) and three strain-hardening stiffness ratios ($\alpha = 0$, 0.02, 0.05). A total of 972 combinations of SDOF systems were considered. The P–Δ effects and cyclic deterioration were not included in the analysis for the purpose of simplicity.

Table 1. ID numbers of different SGMRs used.

ID	PEER-NGA Rec. #	Event, year	M_w	R_{ave}	ID	PEER-NGA Rec. #	Event, year	M_w	R_{ave}
1	953	Northridge, 1994	6.7	13.3	23	848			19.85
2	1602	Duzce, Turkey, 1999	7.1	12.2	24	960	Northridge, 1994	6.7	11.9
3	1602			12.2	25	752	Loma Prieta, 1989	6.9	22.1
4	1787	Hector Mine, 1999	7.1	11.2	26	752			22.1
5	1787			11.2	27	767			12.5
6	169	Imperial Valley, 1979	6.5	22.25	28	767			12.5
7	169			22.25	29	1633	Manjil, Iran, 1990	7.4	12.8
8	174			13	30	1633			12.8
9	174			13	31	721	Superstition Hills, 1987	6.5	18.35
10	953	Northridge, 1994	6.7	13.3	32	721			18.35
11	1111	Kobe, Japan, 1995	6.9	16.15	33	725			11.45
12	1111			16.15	34	725			11.45
13	1116			23.8	35	829	Cape Mendocino, 1992	7	11.1
14	1116			23.8	36	829			11.1
15	960	Northridge, 1994	6.7	11.9	37	1244	Chi-Chi, Taiwan, 1999	7.6	12.75
16	1158	Kocaeli, Turkey, 1999	7.5	14.5	38	1244			12.75
17	1158			14.5	39	1485			26.4
18	1148			12.05	40	1485			26.4
19	1148			12.05	41	68	San Fernando, 1971	6.6	24.35
20	900	Landers, 1992	7.3	23.7	42	68			24.35
21	900			23.7	43	125	Friuli, Italy, 1976	6.5	15.4
22	848			19.85	44	125			15.4

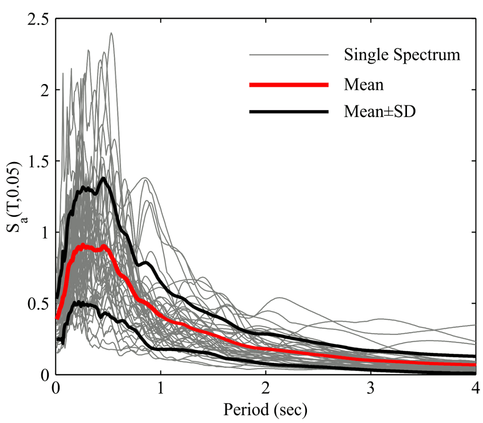

Figure 3. The elastic response spectra for the SGMRs, their mean and the mean ± standard deviation.

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3.3. *PIDA for SDOF systems*

In the third step, IDA curves of SDOF systems were computed. To analyse the SDOF systems, assuming a fixed mass value, the system stiffness can easily be calculated with regard to the selected period of the system. Using ground motion properties and R_y-μ-T equations, consistent with Newmark–Hall inelastic design spectra (Chopra, 2001), the yield strength (F_y), yield deformation (D_y) and other parameters required to perform the analyses of the SDOF systems were computed. Figure 4(a) shows IDA curves and SDOF backbone curve for one of the SDOF systems. The probability density function of the collapse capacity and the corresponding fitted lognormal function are shown in Figure 4(b). The Lilliefors test of normality confirmed the accuracy of the lognormal distribution assumption of the collapse capacity points (p-value = 0.91) (Lilliefors, 1967).

By employing the PIDA, the precedence list for any given system can be calculated. Having the precedence list of SGMRs for each SDOF system, P_{PL} was obtained for 'full data' using all the records for one specific structure (P_{PLf}) and for the 'selected' number of SGMRs based on the precedence list (P_{PLr}). The solution accuracy of the GA, which was used by PIDA, depends on a series of factors such as population size, crossover fraction, mutation function and number of elites. Hence, when employing GA, each system is a separate and independent optimisation problem in which the main factors of the GA should be tuned accordingly, but it was not practical here because there were 972 combinations of SDOF systems. Thus, the required parameters were tuned for one special case of $T = 0.85$ s (mean of the given period range), $\mu = 6$, $\xi = 0.05$ and $\alpha = 2\%$, and then, these parameters were used in the GA optimisation for all SDOF systems. The error in the computed P_{PL} values was defined with respect to P_{PLf} as$(P_{PLf} - P_{PLr})/P_{PLf}$; therefore, a negative error value implies overestimation of P_{PLf}.

Figure 5 shows the 'hazard derivative-fragility product' for two different cases. Figure 5(a) shows an SDOF system with $T = 0.55$ s and different ductility values, whereas Figure 5(b) represents a fixed ductility value equal to six and different SDOF systems periods.

Figure 6 shows the estimation of the collapse capacity distributions for the full data set and the reduced data set for different number of selected SGMRs (3, 6, 9 and 12) on the basis of the improved fitness function along with their respective IDA curves for one of the SDOF system. It is evident that using the improved fitness function can provide good estimate of the distribution while using a limited number of SGMRs.

3.4. P_{PL} computation and the respective errors

As the fourth step, P_{PLf}, P_{PLr} and their respective errors were computed. Figure 7(a and b) shows the P_{PLf} and P_{PLr} for $\xi = 0.05$ and $\xi = 0.07$, respectively, with all other parameters fixed (six SGMRs, $R_{rupture} = 10$ km, $M_w = 6.5$, $\alpha = 0.02$ and $TR = 475$ years). P_{PLf} is shown using surface and P_{PLr} with the

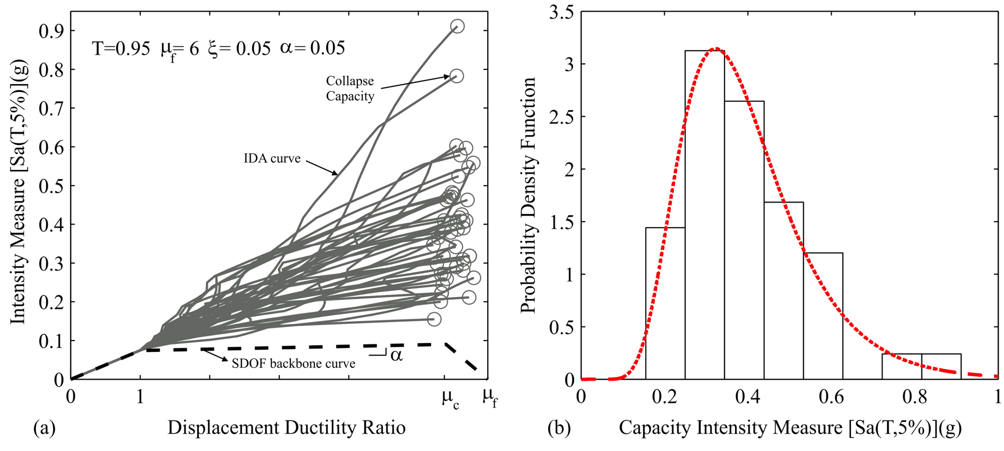

Figure 4. (a) IDA curves for a sample SDOF system with $T = 0.95$ s, $\mu = 6$, $\xi = 0.05$, $\alpha = 0.05$ and (b) the distribution of collapse capacity.

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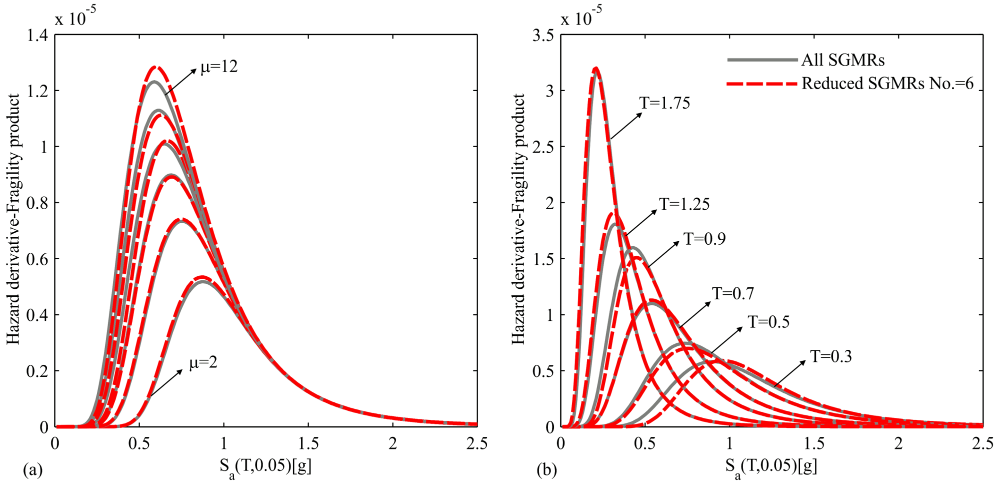

Figure 5. Hazard derivative-fragility product for $M_w = 7$, $R_{rupture} = 10$, $TR = 475$ years: (a) SDOF system for different ductility values with $T = 0.55$ s, $\xi = 0.05$ and $\alpha = 0.05$ and (b) SDOF system for different period values with $\mu = 6$, $\xi = 0.05$ and $\alpha = 0.05$.

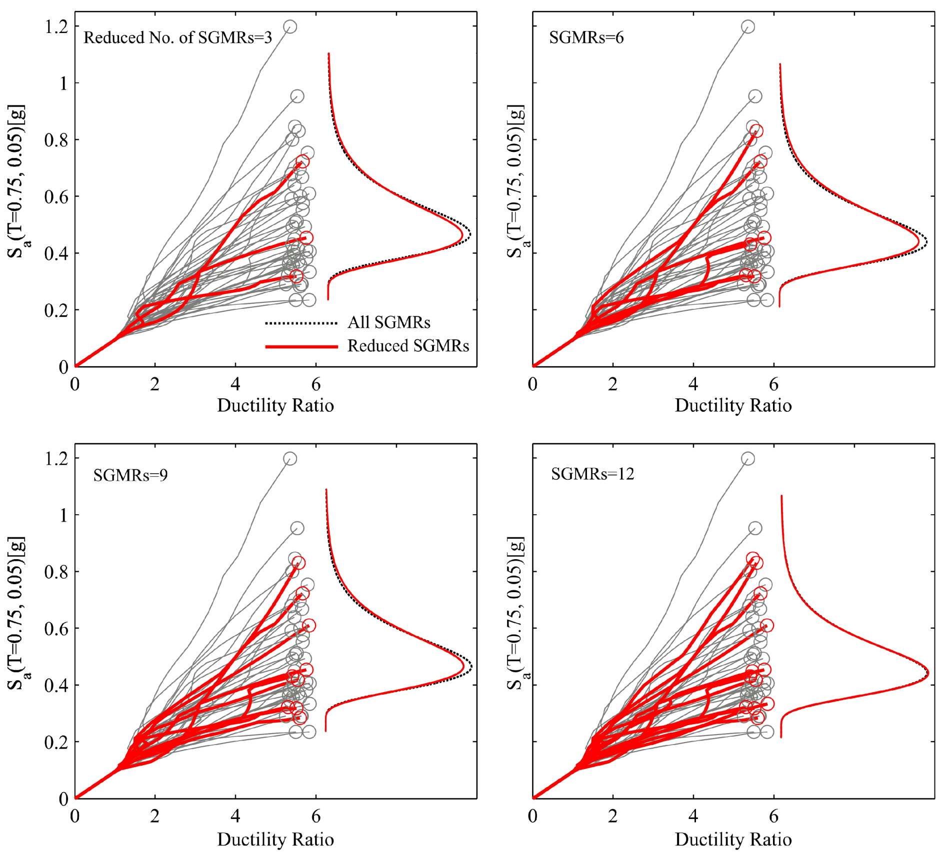

Figure 6. Effect of number of selected SGMRs on probability density function of collapse capacity using improved fitness function for an arbitrary system with $T = 0.75$ s, $\mu = 6$, $\xi = 0.05$ and $\alpha = 0.05$.

mesh. As stated earlier, using a limited number of SGMRs may lead to overestimation or underestimation of P_{PL}. The light regions in Figure 7(a and b) imply the overestimation of P_{PL}, while the dark regions imply that a reduced number of SGMRs has led to an underestimation of P_{PL}. This

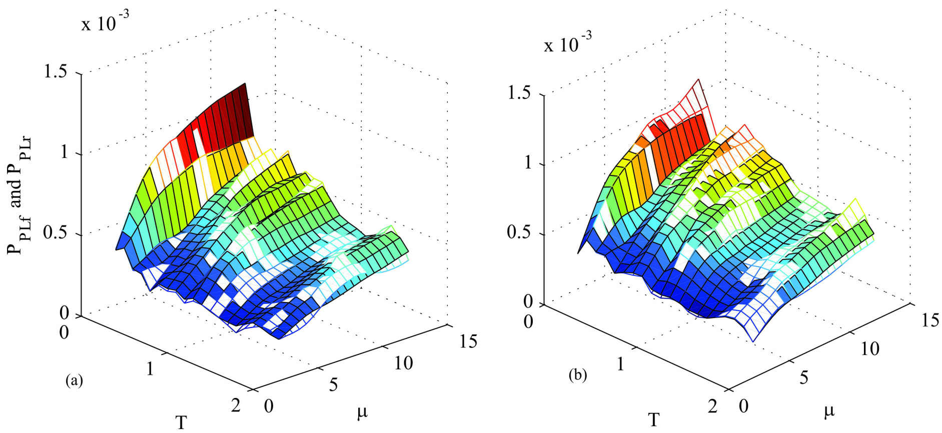

Figure 7. Computed values of P_{PLf} and P_{PLr}: (a) $\xi = 0.05$ and (b) $\xi = 0.07$, respectively. Mesh indicates P_{PLr} and surface shows the P_{PLf}; regions in white means P_{PLr} is overestimating the P_{PLf}.

difference or error was investigated next for the full range of data including all combinations of SDOF systems and hazard conditions (see Figure 2).

Figure 8(a and b) shows the error bar diagram for comparison of error values at 95% confidence level (CL) for different numbers of selected SGMRs and different fitness functions. The error bars represent the mean $\pm$ 1.96$\times$ (standard deviation) of the 14,580 computed error values for each number of SGMRs. It can be seen that at least six SGMRs were needed to be used in the improved method to keep the errors relatively low (less than 15%), but the error range is relatively higher in the case of the original fitness function as shown in Figure 8(a).

3.5. *Statistical analysis for determining the minimum number of SGMRs*

It is worth emphasizing that, as a whole, the mean of error, using any number of SGMRs greater than or equal to six, reaches an appropriate value of less than 6%. However, to determine the existence of meaningful differences in the mean values of P_{PL} errors using different numbers of SGMRs, these groups were compared using repeated measures ANOVA (Davis, 2002). Figure 9 shows the comparison of mean error values employing different numbers of SGMRs. According to this comparison, using nine SGMRs in the improved method would increase the mean value of error to the extent that,

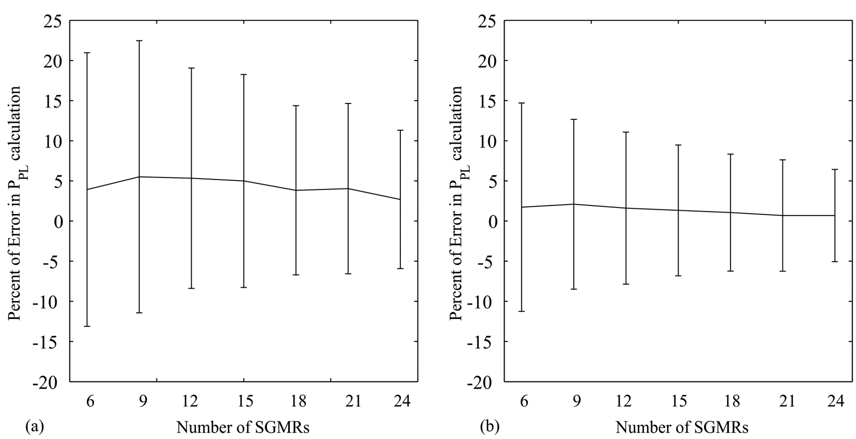

Figure 8. Comparison of error in P_{PL} calculation at 95% confidence level (mean $\pm$ 1.96$\times$ (standard deviation)) for different number of SGMRs: (a) using original fitness function and (b) using improved fitness function.

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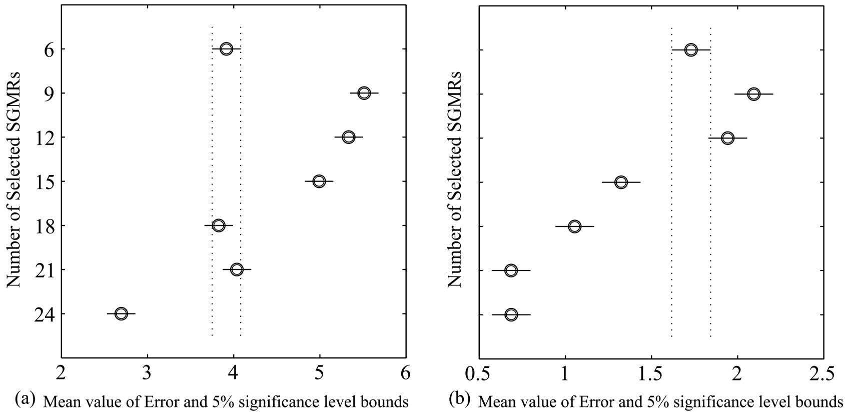

Figure 9. Comparison of mean error value at 5% significance level considering different number of SGMRs: (a) original fitness function and (b) improved fitness function.

at 5% significance, it is considered higher than using six SGMRs. On the other hand, using 12 SGMRs did not make any significant improvement in computing the mean error in comparison with the six SGMRs. Although using more than 12 SGMRs may reduce the mean value of error, the interest of this article is to reduce the number of SGMRs while obtaining good and applicable approximations of the response. Six SGMRs are considered as the minimum and tentatively sufficient number of SGMRs to obtain the appropriate mean results. Considering the original method, same conclusions can be made except that the mean error is higher than that in the improved method using only six SGMRs.

4. APPLICATION OF THE PROPOSED METHOD ON MDOF STRUCTURAL SYSTEMS

In this section, the improved method and the original method (by using both GA and simple optimisation techniques (Azarbakht and Dolšek, 2011)) were employed on three different MDOF systems, namely, a 3-storey, an 8-storey and a 12-storey structure to compare their behaviour. First, general definitions and assumptions are presented, and then they are numerically investigated. Figure 10 shows the steps involved to determine the P_{PL} for the MDOF system.

4.1. General definitions

In order to determine the P_{PL} for MDOF systems, a hazard curve based on the probabilistic seismic hazard analysis has been considered. In this hazard curve, S_a (1 s) for 50% in 50 years, 10% in 50 years and 2% in 50 years equals 0.36 g, 0.59 g and 0.87 g respectively. The site has been located at 20 km from an active fault on stiff soil (V_{s-30} = 350 m/s, National Earthquake Hazards Reduction Program

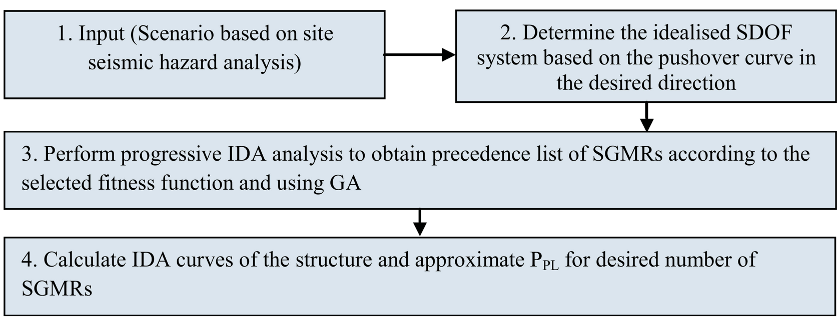

Figure 10. Steps involved in determination of P_{PL} using progressive IDA.

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site class D). It is usually helpful to estimate the hazard especially in the region of interest by a power-law relationship: $H_{Sa} = k_0(s_a)^k$ (Cornell *et al*., 2002). Damping ratio of 5% has been assumed for analyses.

Furthermore, a CL can be computed corresponding to an allowable probability noted as P_0 (Jalayer and Cornell, 2003). In Equation (5), k_x is the standard Gaussian variate with the probability x of not being exceeded and β_U is the dispersion measure representing the total epistemic uncertainty in the IM-based approach.

$$e^{k_x} \leq \frac{P_0}{P_{PL}} e^{-k\beta_U} \tag{5}$$

By solving Equation (5), k_x and the corresponding CL can be computed from a normal distribution table. In calculation of the fragility curves, to determine the probability and the mean annual frequency of collapse, a dispersion of 0.34 has been considered and added to the randomness dispersion computed from IDA analyses to account for modelling uncertainty as suggested by Haselton (Haselton and Deierlein, 2007). The error definition for CL is the same as the error defined previously for P_{PL} computation.

4.2. *Three-storey RC structure*

In this section, a three-storey 3D reinforced concrete structure designed by Fardis (2002) for which a pseudo-dynamic experiment was performed at full scale at the ELSA Laboratory, within the European research project SPEAR ('Seismic performance assessment and rehabilitation of existing buildings') (Negro *et al*., 2004), was investigated. The structure has $T_1 = 0.85$ s, and the idealized period for the corresponding first-mode equivalent SDOF system is 0.92 s. A more detailed explanation of the model and comparison of experimental and numerical results can be found in (Fajfar *et al*., 2006). The nonlinear response-history analyses were performed on the weak (X) direction of the structure. Figure 11 shows the IDA curves, the pushover curve in the X direction and the equivalent SDOF backbone behaviour. The force–displacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve by a transformation factor Γ (Fajfar, 2000).

The original PIDA and improved PIDA along with the simple method (Azarbakht and Dolšek, 2011) were applied to the MDOF test structure on the basis of the first-mode equivalent SDOF system and using the SGMRs database to obtain the precedence list.

Figure 12 shows that even with a small number of SGMRs, 6 out of 44, the improved PIDA can provide a good estimate of the collapse capacity distribution on the basis of the analysis of the first-mode

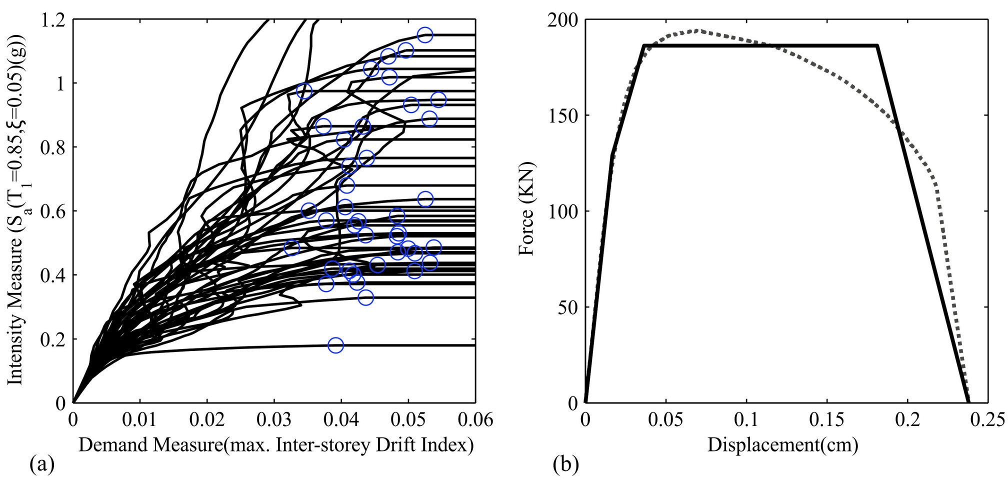

Figure 11. (a) IDA curves and (b) pushover curve in X direction and the equivalent idealized SDOF behaviour.

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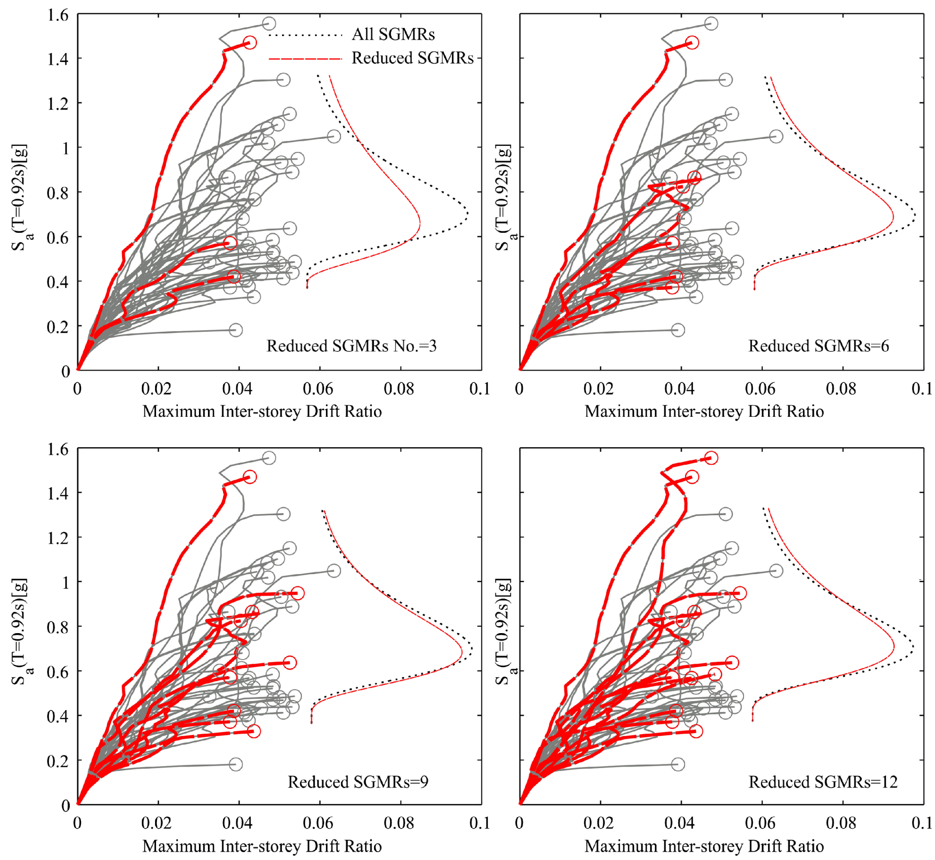

Figure 12. The effect of number of selected SGMRs on the probability density function of collapse capacity using the improved fitness function for the three-storey RC structure.

equivalent SDOF system for the structure studied. Table 2 shows the comparison of obtained results using six SGMRs and different fitness functions. β_R is the dispersion measure representing randomness uncertainty (it is the logarithmic standard deviation of the collapse capacity).

4.3. *Eight-storey modern RC frame*

As the second example in this section, an eight-storey reinforced concrete structure was investigated. The building is 36.5×36.5 m in plan, uses a three-bay perimeter frame system with a spacing of 6.1 m and has a fundamental period (T_1) of 1.71 s designed based on modern building codes. This building is ID 1011 from Haselton and Deierlein (2007). The force–displacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve by a

Table 2. Comparison of obtained results using different fitness functions and six SGMRs for the three-storey RC structure.

Method	Eq. IDs	β_R	$e^{\mu_{\mathrm{Ln}\,S_a(T_1,\mathrm{col})}}$	P_{PL}	Error in P_{PL}	CL%[a]	Error in CL%
Best-estimate	All data set records	0.45	0.6416	0.0104	—	1.0	—
Original PIDA using GA	22, 33, 30, 23, 17, 38	0.56	0.6288	0.0148	−42.42	0.45	55.42
Original PIDA using simple method	22, 33, 30, 27, 13, 17	0.61	0.7154	0.0125	−20.3	0.949	5.71
Improved PIDA using GA	17, 33, 22, 2, 20, 36	0.51	0.67	0.0109	−4.82	1.044	−3.71

[a]$P_0 = 0.0004$ corresponding to 2% in 50 years hazard level.
[b]Without considering ε effects.

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transformation factor Γ (Fajfar, 2000). The first-mode force distribution has been considered to perform pushover analysis of this system. Figure 13 shows IDA curves, the first-mode pushover curve and the equivalent idealized SDOF behaviour. The damping ratio was selected to be 5%. Table 3 shows the comparison of obtained results using six SGMRs and different fitness functions for the previously defined hazard level. It should be noted that in the current calculations, the effects of ε has not been incorporated, which could significantly increase CL in this case. Here, comparative aspects of the methods were of interest. Including ε would lead to more collapse capacity and less dispersion and could increase the associated CL.

4.4. *12-storey modern RC frame*

In the third example, a 12-storey reinforced concrete structure was examined. The building dimensions are the same as the previous example, and it has a fundamental period (T_1) of 2.01 s. This building is ID 1013 from (Haselton and Deierlein, 2007). For this building, the pushover analysis has been performed using the first-mode force distribution. The force–displacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve by a transformation factor Γ (Fajfar, 2000). Figure 14 shows IDA curves, the first-mode pushover curve and the equivalent idealized SDOF behaviour for the 12-storey RC frame. Table 4 summarizes the obtained results for the considered hazard level. Figure 15 shows the comparison of fragility functions and hazard derivative-fragility product using six SGMRs and different fitness functions.

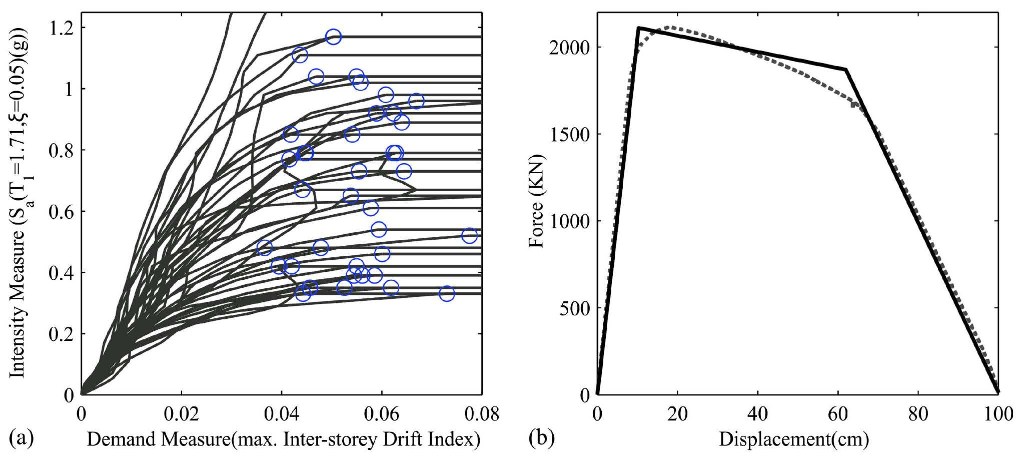

Figure 13. (a) IDA curves and (b) first-mode pushover curve and the equivalent idealized SDOF behaviour.

Table 3. Comparison of obtained results using different fitness functions and six SGMRs for the eight-storey modern RC frame.

Method	Eq. IDs	β_R	$e^{\mu_{\mathrm{Ln}\,S_a(T_1,\mathrm{col})}}$	P_{PL}	Error in P_{PL}	CL%[b]	Error in CL%
Best-estimate	All data set records	0.44	0.66	0.0011	—	27.7	—
Original PIDA using GA	35, 32, 37, 38, 12, 34	0.52	0.6412	0.0016	−47.96	20.17	27.17
Original PIDA using simple method	14, 18, 27, 9, 6, 13	0.504	0.63	0.0015	−46	20.2	26.86
Improved PIDA	12, 20, 27, 11, 14, 30	0.431	0.64	0.0011	−5.5	26.5	4.34

[a]$P_0 = 0.0004$ corresponding to 2% in 50 years hazard level.
[b]Without considering ε effects.

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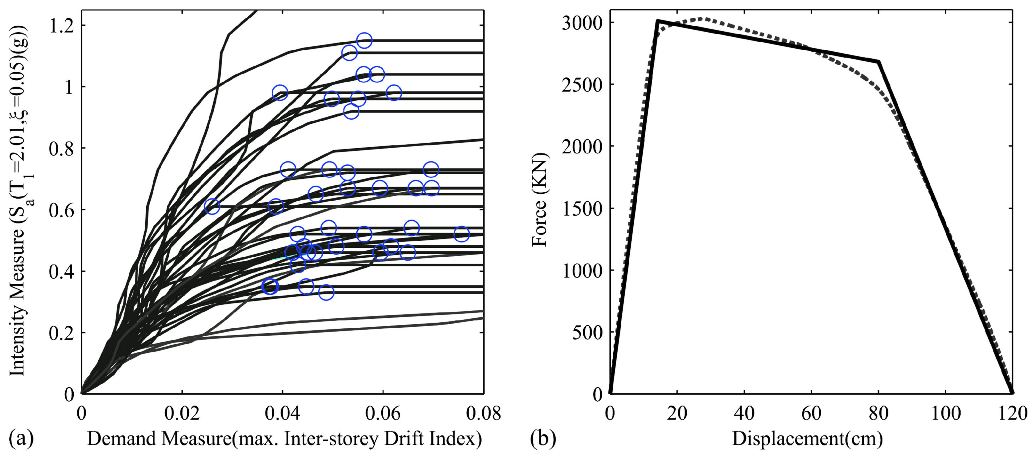

Figure 14. (a) IDA curves and (b) first-mode pushover curve and the equivalent idealized SDOF behaviour.

Table 4. Comparison of obtained results using different fitness functions and six SGMRs for the 12-storey modern RC frame.

Method	Eq. IDs	β_R	$e^{\mu_{\mathrm{Ln}\,S_a(T_1,\mathrm{col})}}$	P_{PL}	Error in P_{PL}	CL%[b]	Error in CL%
Best-estimate	All data set records	0.396	0.6011	0.0006	—	41.83	—
Original PIDA using GA	6, 4, 21, 42, 30, 1	0.33	0.6834	0.0003	50.11	57.75	−38.05
Original PIDA using simple method	6, 4, 21, 30, 41, 27	0.32	0.6767	0.0003	50.25	57.85	−38.3
Improved PIDA	21, 43, 4, 15, 24, 40	0.365	0.5812	0.0006	1.34	42.42	−0.5

[a]$P_0 = 0.0004$ corresponding to 2% in 50 years hazard level.
[b]Without considering ε effects.

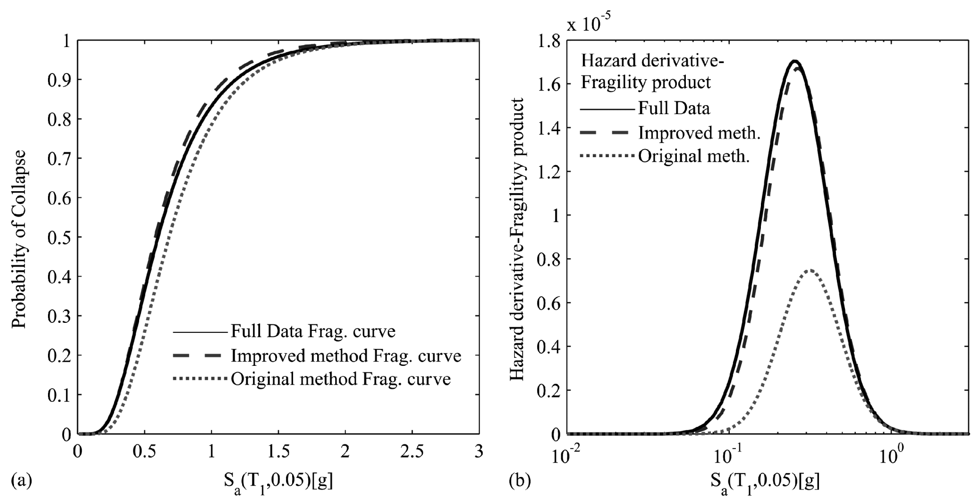

Figure 15. (a) Comparison of failure fragility curve using six SGMRs and different fitness functions and (b) comparison of hazard derivative-fragility product employing six SGMRs and different fitness functions.

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5. CONCLUSION

An improved version of the PIDA to estimate the annual probability of failure of structures has been proposed. This method offers much less computational effort, which is very important as the structure grows larger, and makes it possible to explicitly consider the randomness of the input SGMRs. It also provides a good approximation of P_{PL} value. The first-mode equivalent SDOF system for a given structure obtained by the pushover analysis and the GA optimisation technique was utilized to accurately determine the failure fragility curve and the corresponding annual frequency of failure.

A sensitivity analysis using results of an SDOF database with different variables revealed that, at least for the selected SGMRs database and within the given assumptions, a good approximation for the probability of failure can be obtained by using only six SGMRs. The 95% error bound was between +15% and −11%. Analysis of MDOF systems showed that this method could very effectively predict the fragility curve and the annual probability of failure of these structures using a limited number of SGMRs. Because the proposed method considers the first-mode equivalent SDOF system of the structure, for the time being, it can only be used for the structures with a first dominant mode. It is worth mentioning that in the case of 'ductile' structures and structures with higher first-mode periods, considering ε (FEMA P695, 2009) would lead to more appropriate estimation of structural capacity but here only comparative aspects of different fitness functions were of interest.

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