# Prediction of the median IDA curve by employing a limited number of ground motion records

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#### SUMMARY

A methodology has been proposed which can be used to reduce the number of ground motion records needed for the reliable prediction of the median seismic response of structures by means of incremental dynamic analysis (IDA). This methodology is presently limited to predictions of the median IDA curve only. The reduction in the number of ground motion records needed to predict the median IDA curve is achieved by introducing a precedence list of ground motion records. The determination of such a list is an optimization problem, which is solved in the paper by means of (1) a genetic algorithm and (2) a proposed simple procedure. The seismic response of a simple, computationally non-demanding structural model has been used as input data for the optimization problem. The presented example is a three-storey-reinforced concrete building, subjected to two sets of ground motion records, one a free-field set and the other a near-field set. It is shown that the median IDA curves can be predicted with acceptable accuracy by employing only four ground motion records instead of the 24 or 30 records, which are the total number of ground motion records for the free-field and near-field sets, respectively. Copyright © 2007 John Wiley & Sons, Ltd.

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## 1. INTRODUCTION

Incremental dynamic analysis (IDA) [1] is a general parametric analysis method for the estimation of seismic demand and capacity for different levels of seismic intensity measure (IM), based on the ground motion records within a set defining the earthquake scenario. Such an approach requires a huge computational effort, especially due to the many ground motion records involved in the



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analysis. In order to reduce the computational effort, a number of different approximate methods have recently emerged. Most practical, i.e. approximate methods for IDA analysis involved the replacement of nonlinear dynamic analysis by a combination of pushover analysis of a multi-degree-of-freedom (MDOF) model and nonlinear dynamic analysis of single-degree-of-freedom (SDOF) model [2–5]. On the other hand, many researchers have tried to reduce the dispersion in IDA results (e.g. [6–10]). Typically, a reduction in dispersion by a factor of two means that four times fewer records are needed to gain the same confidence [11]. The goal of these studies is the improvement of IM efficiency in order to reduce the dispersion in the nonlinear response. In addition to the IM efficiency, the sufficiency of IM can increase the robustness in the results obtained from IDA analysis [12].

Although dispersion in seismic response is an important result, it is often more important to quantify the median response, e.g. the median IDA curve. However, ground motion records for nonlinear dynamic analysis, and for IDA analysis, are chosen based on magnitude, distance, and site conditions (e.g. [13, 14]), without any explicit consideration of structural characteristics. The selection of ground motion records requires the calculation of seismic response for all ground motion records that are selected as representative of an earthquake scenario. It would, therefore, be useful to know in advance which ground motion records are the best representatives for the prediction of a median seismic response or, for example, to identify critical ground motion records that are to be used in physical testing [15]. Such approaches could significantly decrease the number of ground motion records needed for the sufficiently accurate prediction of seismic response.

With the proposed methodology, an attempt has been made to reduce the number of ground motion records for predicting the median IDA curve. For this purpose, a precedence list of ground motion records has been introduced, which can be determined by means of either a genetic algorithm (GA) or a proposed simple procedure. The methodology is illustrated using an example of a three-storey-reinforced concrete building subjected to a free- and near-field set of ground motion records.

# 2. METHODOLOGY

The aim of the methodology is to decrease the number of ground motion records needed for the prediction of a median IDA curve. In addition to the MDOF model, which is employed in the IDA analysis [1], the advantages of the simple model (e.g. the SDOF model), which is not computationally demanding, are taken into account. Such an approach is employed in many other approximate methods (e.g. [2, 3]). These methods use the response of the simple model, in combination with the pushover analysis, to predict the seismic response of the MDOF model. However, the methodology described employs the simple model only to predict the precedence list of ground motion records. Single-record IDA curves are then calculated, step by step using the MDOF model from the precedence list of ground motion records until acceptable tolerance for the median IDA curve is reached. The main steps of the methodology are presented in Figure 1, and can be described as follows:

1. Select a set of ground motion records based on the earthquake scenario. This is the same step as in an IDA analysis. The number of records within the given set can, if so desired, be high, since, when using the methodology, there is no need to compute the seismic response of the MDOF model for all records in order to obtain a good prediction of the median IDA curve.



Figure 1. The main steps in the proposed methodology.

- 2. Create a MDOF mathematical model that can be used for the simulation of the realistic seismic response of the structure under investigation.
- 3. Define a simple mathematical model, e.g. a SDOF model. This model should be a good representative of the linear and nonlinear characteristics of the MDOF mathematical model, yet simple enough for it to be possible to perform a large number of nonlinear time-history analyses, without the need for very time-consuming calculations.
- 4. Compute single-record IDA curves for the simple model, for all the ground motion records within the given set. Because of the simplicity of the chosen simple model, this should not be a time-consuming task.
- 5. Based on the results obtained in step 4, arrange the ground motion records within the given set in order to obtain a good precedence list. This is an optimization problem, which is explained in the next section. The objective of the optimization is to minimize the differences between the 'original' and the 'selected' median IDA curves. The 'original' median IDA curve is obtained from all the single-record IDA curves (step 4), whereas the 'selected' median IDA curves are obtained only for the first *s* ground motion records from the precedence list, where *s* is the number of 'selected' ground motion records. The number of median IDA curves, based on the *s* ground motion records, is thus equal to the number of ground motion records in the set being used.
- 6. Compute a single-record IDA curve for the MDOF model, starting with the first record from the precedence list. After computation of the single-record IDA curves for the *s*th record from the precedence list (where *s* is a number greater than or equal to two), compute the 'selected' median IDA curve and compare it with the 'selected' median IDA curve obtained from the (s 1)th records.

- 7. Repeat step 6 until the difference between the 'selected' median IDA curves, determined for the *s*th and (s 1)th records, is less than the acceptable tolerance, and then stop performing the IDA analysis on the MDOF model.
- 8. The 'selected' median IDA curve, calculated from the *s* single-record IDA curves with dispersion responses based on SDOF IDAs, can be used for further seismic performance assessment.

The described procedure can significantly reduce the number of nonlinear time-history analyses needed to predict the median IDA curve with sufficient accuracy. However, the efficiency of the procedure depends on the ability of the simple model to predict the damage measure of the MDOF model, as well as on the ability of the optimization algorithm to find the best precedence list of ground motion records. The median IDA curve, obtained from the described procedure by employing a limited number of ground motion records, is usually a good approximation to the 'original' median IDA curve for the MDOF model, which is calculated from all the single-record IDA curves.

The choice of the simple mathematical model is important, since the precedence list of ground motion records is obtained from the IDA analysis on the simple model. It is, therefore, desirable that IDA curves determined by using the simple model do not differ significantly from the IDA curves determined by using the MDOF model, although the problem is constrained by the fact that analyses with the simple model should not be time consuming. Note that the simple model cannot capture the failure mechanisms that are present in the more realistic MDOF model. However, the ground motion records, which can be used to predict a good median IDA curve for the simple model, are just good representatives for the prediction of the median IDA curve for the MDOF model.

Also note that the procedure can be easily applied to other problems, and not just to the problem of minimizing the number of records for the sufficiently accurate prediction of the median IDA curve. For example, the procedure can be applied for the selection of a certain number of records for a purpose of an experiment as well as for a particular design purpose. For the latter case, many codes recommend using a certain number of records for the prediction of the most critical actions and/or a different number of records (usually more) for the prediction of the mean or median response. In this case, the described approach can significantly reduce the bias in the seismic response which is present because of the limited number of ground motion records prescribed for nonlinear dynamic analyses.

#### 3. PRECEDENCE LISTS OF GROUND MOTION RECORDS

A precedence list of ground motion records can, in general, be determined by many different optimization techniques. Although this is quite a simple optimization problem, it is better to use optimization techniques that are not gradient based. One such technique, which is used in this study, is the GA. The advantage of the GA is that there is a high probability that the fitness (objective) function converges to near the global minimum of the problem and not to a local one, which is not always the case for gradient-based searching techniques. The fitness function, which quantifies the adequacy of the solution, is always problem dependent (Section 3.1). However, it is important to define this function properly in order to obtain a good solution. The GA, whose elements are briefly described in Section 3.2, is capable of minimizing only the fitness function.

In addition to the GA, a simple procedure for minimizing the fitness function is also proposed (Section 3.3). The benefit of such an approach is the simplicity of the solution.

It should be emphasized that the precedence list of ground motion records is calculated based on the IDA analysis of a simple model (e.g. a SDOF model). There is, therefore, no guarantee that such a precedence list of ground motion records is also an optimal solution for the MDOF model. Only from the single-record IDA curves, computed with MDOF model, it is possible to measure the quality of the solution. Because the single-record IDA curves are calculated gradually from the precedence list of ground motion records and it is desirable to calculate as few single-record IDA curves as possible on the MDOF model, an acceptable tolerance for the median IDA curve, which is explained in Section 3.4, is proposed.

#### 3.1. The fitness function

It is not a trivial matter to define the fitness function of the studied problem. Basically, it is desirable to capture the best measure for recognizing the difference between the 'original' and 'selected' median IDA curve (Section 2, step 5). A good measure for this purpose is the normalized area between the two median IDA curves. The part that is able to minimize the dispersion is also included. The fitness function is, therefore, defined as the sum of two parts divided by the number of all ground motion records (n):

$$Z = \frac{1}{n} \times \sum_{s=1}^{n} V(s) = \frac{1}{n} \times \sum_{s=1}^{n} (\text{Error Term}(s) \times (1 + \text{Dispersion Term}(s)))$$
(1)

where the difference between the 'original' and 'selected' median IDA curve is quantified by the error term (Equation (2)), and the dispersion of the *s* selected single-record IDA curves is quantified by the dispersion term (Equation (3)). Note, as explained in Section 2, that the 'original' median IDA curve is obtained from all the single-record IDA curves, whereas the 'selected' median IDA curve is obtained for just the first *s* ground motion records from the precedence list. The number of 'selected' median IDA curves, is therefore, the same as the number of all ground motion records (*n*) within a given set of ground motion records. In order to determine the precedence list of ground motion records, summation of the so-called 'partial' fitness function V(s) (Equation (1)) needs to be performed. The 'partial' fitness function V(s) is introduced only for the purpose of an easier explanation of the simple procedure for determining the precedence list of ground motion records (Section 3.3). However, minimizing the 'partial' fitness function means the selection of *s* ground motion records, which are the best representatives of the 'original' median IDA curve.

The normalized area, expressed as percent, between the 'original' and the 'selected' median IDA curve can be calculated as

Error Term(s) = 
$$100 \times \frac{\int_0^{\max(DM_c^{\text{or}}, DM_c^{\text{or}})} |\Delta IM(s)| \, dDM}{\int_0^{DM_c^{\text{or}}} IM_{\text{or}} \, dDM}$$
 (2)

where DM is the damage measure, IM is an intensity measure for the IDA analysis,  $\Delta IM(s)$  is the difference in the IM corresponding to the 'original' and 'selected' median IDA curve, DM<sup>s</sup><sub>c</sub> is the maximum DM that corresponds to the capacity point in the case of the 'selected' median IDA curve (Figure 2), DM<sup>or</sup><sub>c</sub> is the maximum DM that corresponds to the capacity point in the case of the 'original' median IDA curve (Figure 2), and IM<sub>or</sub> is the intensity measure of the 'original' median IDA curve (Figure 2). The parameter  $\Delta IM(s)$  depends on the *s* ground motion records

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Figure 2. Schematic definition of the parameters for calculation of the ErrorTerm (*s*), Equation (2), based on four possible conditions of the 'original' median IDA curve and the 'selected' median IDA curve, which is determined based on *s* selected ground motion records.

which are employed to determine the 'selected' median IDA curve, and also depends on the DM, as schematically shown in Figure 2. Different possibilities of the relationship between the 'original' and 'selected' median IDA curves and the explained parameters of Equation (2) are presented in Figure 2.

The dispersion term in the fitness function Z (Equation (1)) is defined as

Dispersion Term(s) = 
$$\frac{1}{IM_c^s} \times \int_0^{IM_c^s} Dispersion(s) dIM$$
 (3)

where Dispersion(s) is the dispersion measure, usually a standard deviation of natural logarithms,  $IM_c^s$  is the intensity measure that corresponds to the capacity point in the case of the 'selected' median IDA curve (Figure 2). Note that dispersion not only depends on the IM but also on the number of selected ground motion records. In this study, dispersion was calculated from the 16% and 50% fractile values, assuming a lognormal distribution of the damage measure at a given IM. The first part of the expression on the right-hand side of Equation (3) averages the dispersion for the IM.

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#### 3.2. GA-based determination of ground motion records precedence list

The concept of a GA was proposed in 1975 at Michigan University by Holland [16] and developed by Goldberg [17]. GAs are robust in producing near-optimal solutions, with a high degree of probability that the global optimum will be obtained [18]. In structural and earthquake engineering, over the past decade GAs have been used for many applications, for example, for the optimization of nonlinear structures [19] and for the selection and scaling of ground motion records [20]. Only a short description of each element of the GA, with the corresponding assumptions for this problem (needed in order to obtain satisfactory results), is given here.

The fitness function is always the target for the algorithm, which should be minimized or maximized. This function has been introduced in detail in Section 3.1. Once the fitness function is defined, the GA randomly generates an initial population of 50 individuals, where each individual is an  $1 \times n$  array that represents all the ground motion record IDs in a specified set of ground motion records. A certain number of the best individuals (10% of individuals, i.e. five precedence lists) are selected as elites for passing into the next generation without any changes. In each new generation, some of the new individuals are generated by means of a crossover function. In this study, a scattered crossover pattern was used. This crossover function creates a random binary vector (which is  $1 \times n$ ) and selects the genes (in this case ground motion record IDs) where the value of this vector is 1 from the first parent and zero from the second parent. It combines the genes from both parents to form a new child [21]. The crossover fraction was chosen to be 0.65. This means that 65% of 45 individuals who have fitter fitness values, other than elite children, are used for parents. Hence, the algorithm rounds  $0.65 \times 45 = 29.3$  to 29 to get the number of crossover children. In each new generation, 16 new individuals (significantly fewer than the individuals from the crossover function) are generated by means of a mutation function. This function is a necessary part of the GA, and prevents it from converging to a local optimum. For this purpose, the Gaussian mutation function was selected [21], which randomly changes some of the genes sequences in individuals to produce new individuals who were probably not present in the initial population. In order to apply all the above concepts, a GA, utilizing the GA toolbox in MATLAB 7.3.0.267 (R2006b) software, was used to solve the problem [21].

# 3.3. A simple procedure for determining the precedence list of ground motion records

A simple procedure for determining the precedence list of ground motion records is introduced based on the assumption that the minimum of the fitness function Z (Equation (1)) can be found by gradual minimization of the 'partial fitness function' V(s) at each step of s, the selected number of ground motion records. This assumption makes possible very simple and rapid determination of the precedence list, as explained in the following steps:

- 1. For s = 1, calculate the partial fitness function V(1) for each ground motion record from the given set of ground motion records. Clearly, there are *n* partial fitness functions V(1), where *n* is the total number of ground motion records in the given set. The minimum value of V(1) defines the ground motion record, which is the first on the precedence list. This means that, in the first step, the single-record IDA curve that has the minimum deviation from the median IDA curve is the target IDA curve.
- 2. Increase s by 1 and calculate the partial fitness function V(s) for n s + 1 combinations of s selected ground motion records. These combinations are determined from the fact that s 1 ground motion records are already fixed in the precedence list, so that only n s + 1

ground motion records are candidates for the *s*th place on the precedence list of ground motion records. As in step 1, the minimum value for V(s) (out of n - s + 1 values) defines the ground motion record at the *s*th place on the precedence list.

3. Continue step 2 until s = n.

The described procedure is easy to program, and much less time consuming than the solution determined by GA. An additional advantage of the simple procedure, in comparison with GA, is the ease of interpretation of the results, i.e. the precedence list, which can be ascribed to the philosophy of the simple procedure used.

#### 3.4. Tolerance for the median IDA curve of the MDOF model

Although the precedence list of ground motion records for determining the median IDA curve is known, it is not known in advance how many ground motion records are needed for adequate evaluation of the median IDA curve of the MDOF model. Some criteria may need to be defined for decision-making about the sufficient number of ground motion records for prediction of the median IDA curve. For this purpose, the computation of single-record IDA curves based on the MDOF model can be started gradually from the precedence list. After the calculation of the single IDAs for *s* records from the precedence list (where *s* is greater than or equal to two), compute the 'selected' median IDA curve and compare it with the 'selected' median IDA curve obtained from the s - 1 records. This criterion is a good measure for tracing the step-by-step changes in the median IDA curve. The mathematical form of this criterion, which is called a tolerance function, is shown in the following equation:

Tolerance(s) = 
$$100 \times \frac{\int_{0}^{\max(DM_{c}^{s-1}, DM_{c}^{s})} |\Delta IM_{s,s-1}| dDM}{\int_{0}^{DM_{c}^{s-1}} IM_{s-1} dDM}$$
 (4)

where DM is the damage measure,  $IM_{s-1}$  is the intensity measure for the s-1 selected median IDA curve,  $\Delta IM_{s,s-1}$  is the difference in the IM of the s and s-1 selected median IDA curves, and  $DM_c^s$  and  $DM_c^{s-1}$  are, respectively, the maximum DMs which are usually defined by the capacity point on the 'selected' median IDA curve, which are determined based on sth or (s-1)th ground motion records. Equation (4) can be explained as the ratio between two areas. The area that represents the difference between the two 'selected' median IDA curves, one determined for s-1 and the other for s ground motion records, is divided by the area under the 'selected' median IDA curve determined for s-1 ground motion records.

#### 4. EXAMPLE

In order to demonstrate the applicability of the proposed methodology, a precedence list of ground motion records has been determined for the median IDA curve (computed by the counting method, e.g. as explained in [3]) corresponding to a three-storey-reinforced concrete frame building. Precedence lists of ground motion records were determined for two sets of ground motion records by employing both of the optimization techniques presented in Section 3. The IM selected in the example was the spectral acceleration at the period of the equivalent SDOF model, which was introduced as a good representative of the simple model. The top displacement and maximum inter-storey drift of the building were chosen as the damage measures. The results are presented



Figure 3. The elevation and plan view of the SPEAR building, showing typical reinforcement details.

in terms of 'selected' median IDA curves, and compared with the 'original' median IDA curve. Based on the acceptable tolerance, the number of ground motion records needed for sufficiently accurate prediction of the median IDA curve is proposed.

The IDA analysis has been performed for the SDOF as well as for the MDOF model simply by assuming the constant increment of 0.01g for the IM. The IDA analysis was performed until the dynamic instability is obtained. Note that a small increment for IM was assumed in order to eliminate the errors, which may appear from the interpolation between discrete points on the IDA curve. Obviously, efficient (minimum) number of points for each IDA curve (equals to the number of nonlinear dynamic analysis for each record) is still open to be investigated.

# 4.1. The test structure and the mathematical model

The test structure (referred to in the following as the SPEAR building) is a three-storey asymmetricreinforced concrete frame building, for which a pseudo-dynamic experiment was performed at full scale at the ELSA Laboratory, within the European research project SPEAR ('Seismic performance assessment and rehabilitation of existing buildings') [22]. This building was designed for gravity loads only. The elevation and plan view of the SPEAR building are shown in Figure 3, together with details about the typical reinforcement in its beams and columns. The so-called post-test mathematical model [23] created within the OpenSees program [24] was employed for the analyses performed in this study. This mathematical model consists of beam and column elements whose flexural behaviour was modelled by one-component lumped plasticity elements, consisting of an elastic beam and two inelastic rotational hinges (defined by the moment–rotation relationship).

#### A. AZARBAKHT AND M. DOLŠEK

A more detailed explanation of the model and a comparison with the experimental results can be found in [23].

For simplicity, the nonlinear dynamic analyses were performed by subjecting the structure to loads in the weak X direction only (Figure 3). For this direction, the ratio between the maximum base shear, obtained from the pushover analysis (Figure 5), and the weight of the building amounted to only about 0.1.

# 4.2. Ground motion records

Two sets of ground motion records were used in the study. The first set consisted of 24 free-field ground motion records, selected from Reference [25]. The earthquake moment magnitudes  $M_w$  for the selected records ranged from 5.5 to 6.5. The second set consisted of 30 near-field ground motion records that were recorded under forward directivity conditions, and for which the moment magnitudes ranged from 6.5 to 6.9. Both the first and second sets of ground motion records were recorded on NEHRP  $S_A$  or  $S_B$  and  $S_D$  or  $S_C$  sites [26], respectively, and were uniformly processed by Walter Silva for the PEER Strong Ground Motion Database [25]. The list of records is presented in Table I, and the acceleration spectra corresponding to both sets of ground motion records are presented in Figure 4.

#### 4.3. IDA analysis for a simple mathematical model

The simple mathematical model is introduced by a SDOF model, which is based on the results of pushover analysis. Pushover analysis of the MDOF model was performed for the X direction only, since the mathematical model of the test structure, too, was subjected only to ground motion in the X direction. The load pattern employed in the pushover analysis corresponded to the dominant mode shape in the X direction. The selected pushover curve and the idealized pushover curve are presented in Figure 5(a).

The SDOF model was then defined based on the approach presented in [27]. The forcedisplacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve by a transformation factor  $\Gamma$ , which in this example, is equal to 1.26. The period of the SDOF model is 0.92. In order to simulate the hysteretic behaviour of the MDOF model, the same hysteretic rules were used for the SDOF model. The force-displacement envelope and the hysteretic behaviour of the SDOF model are presented in Figure 5(b).

In order to obtain the input data needed to define the precedence list of ground motion records, IDA analysis was performed for the SDOF model by applying both sets of ground motions. The single-record IDA curves and the corresponding median IDA curves are presented in Figure 6. The time needed for the IDA analysis performed on the SDOF model is approximately the same as the time needed for one nonlinear time-history analysis of the MDOF model.

# 4.4. Precedence lists of ground motion records determined on the basis of IDA analysis for the simple model

The precedence lists of ground motion records were determined for the free- and near-field sets of ground motion records by employing the GA and the simple procedure, which have been described in Sections 3.2 and 3.3, respectively. The input data for determining the precedence list are single-record IDA curves (Figure 6), the corresponding ID numbers of the ground motion records (Table I), and median IDA curves (Figure 6).

		Table I. The free- and near-fiel	ld sets	of ground motion recor	ds.		
<b>Free-fiel</b>	ld s	st of ground motion records		Near-field s	et of	ground motion records	
Event, year, M <sub>w</sub>	Ð	Station	R	Event, year, $M_{\rm W}$	Θ	Station	R
Morgan Hill, 1984, 6.2	1	Gilroy Array #1/G01230	16.2 In	nperial Valley, 1979, 6.5	25	Brawley Airport	8.5
)	0	Gilroy Array #1/G01320	16.2		26	EC County Center FF	7.6
Coyote Lake, 1979, 5.7	ŝ	San Juan Bautista/ SJB213	17.9		27	EC Meloland Overpass FF	0.5
	4	San Juan Bautista/SJB303	17.9		28	El Centro Array #1	15.5
	S	SJB Overpass, Bent/SJ3067	19.2		29	El Centro Array #4	4.2
	9	SJB Overpass, Bent/SJ3337	19.2		30	El Centro Array #5	1.0
	2	SJB Overpass, Bent/SJ5067	19.2		31	El Centro Array #6	1.0
	×	SJB Overpass, Bent/SJ5337	19.2		32	El Centro Array #7	0.6
Imperial Valley, 1979, 6.5	6	Cerro Prieto/H-CPE147	23.5		33	El Centro Array #8	3.8
	10	Cerro Prieto/H-CPE237	23.5		34	El Centro Array #10	8.6
	11	Parachute Test Site/H-PTS225	14.0		35	El Centro Array #11	12.6
	12	Parachute Test Site/H-PTS315	14.0		36	El Centro Differential Array	5.3
	13	Superstition Mtn Camera/H-SUP045	26.0		37	Westmorland Fire Sta	15.1
	14	Superstition Mtn Camera/H-SUP135	26.0		38	Parachute Test Site	14.2
Livermore, 1980, 5.8	15	Antioch—510 G St/A-ANT270	20.8	Superstition	39	El Centro Imp. Co. Cent	13.9
	16	Antioch—510 G St/A-ANT360	20.8	Hills (B), 1987, 6.7	40	Westmorland Fire Sta	13.3
	17	Fremont-Mission San Jose/A-FRE075	33.1		41	Parachute Test site	0.7
	18	Fremont-Mission San Jose/A-FRE345	33.1	Loma Prieta, 1989, 6.9	42	Saratoga-W Valley Coll.	13.7
Morgan Hill, 1984, 6.2	19	Corralitos/CLS220	22.7	Northridge, 1994, 6.7	43	Canyon Country-W Lost Cany	13.0
1	20	Corralitos/CLS310	22.7	1	4	Jensen Filter Plant	6.2
	21	Gilroy Gavilan Coll/GIL067	16.2		45	Newhall—Fire Sta	7.1
	22	Gilroy Gavilan Coll/GIL337	16.2		46	Rinaldi Receiving Sta	7.1
Parkfield, 1966, 6.1	23	Cholame #12/C12050	14.7		47	Sepulveda VA	8.9
	24	Cholame #12/C12320	14.7		48	Sun Valley—Roscoe Blvd	12.3
					49	Sylmar—Converter Sta	6.2
					50	Sylmar-Converter Sta East	6.1
					51	Sylmar—Olive View Med FF	6.4
					52	Arleta—Nordhoff Fire Sta	9.2
					53	Newhall—W. Pico Canyon Rd.	7.1
					54	Pacoima Dam (downstr)	8.0
<i>Note:</i> $M_{\rm W}$ is the moment n	nagı	uitude and R is the closest distance to the	e fault ru	ıpture (given in km).			

# PREDICTION OF THE MEDIAN IDA CURVE

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2411



Figure 4. The elastic response spectra (5% damping) for the free- and near-field sets of ground motion records.



Figure 5. The pushover curve, the idealized base shear-top displacement relationship (a) and the force-displacement relationship for the SDOF model, with typical hysteretic behaviour (b).

The precedence list of the ground motion records is obtained by rearranging the ID numbers of the ground motion records (Table I) in order to minimize the fitness function (Equation (1)) presented in Section 3.1. In the case of the GA, minimization of the fitness value as a function of the number of generations is shown in Figure 7. In addition to the minimum value of the fitness function, a mean fitness value of the generation is also shown. The mean value is obtained as the mean fitness value within one generation. Note that one generation consists of 50 individuals (Section 3.2), where each individual represents one 'candidate' for the precedence list of ground motion records.

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Figure 6. The single-record IDA curves and the derived median IDA curves, corresponding to the SDOF model, for the free- and near-field set of ground motion records.



Figure 7. The mean and minimum fitness values in each generation for the free- and near-field sets of ground motion records.

The precedence lists of ground motion records are presented in Table II with the ID numbers of the ground motion records presented in Table I, for both sets of ground motion records and for both optimization techniques. Six and four, respectively, out of the first six records are the same in the precedence list for the free- and near-field sets, in the case of both optimization techniques. For example, the 'selected' median IDA curve for the first four ground motion records from the precedence list, which was obtained by using the GA, is presented in Figure 8 and compared with the 'original' median IDA curve. Very good correlation between the two median IDA curves is observed for the free- and near-field sets of ground motion records.

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Figure 8. The single-record IDA curves for SDOF model, showing those corresponding to the first four selected ground motion records, the original median IDA curve, and the selected median IDA curve. The results are shown for the GA optimization technique and for both sets of ground motion records.



Figure 9. The ErrorTerm, (Equation (2)), *versus* the number of selected ground motion records for the SDOF model, presented for both optimization techniques and for both sets of ground motion records.

The ErrorTerm, see Equation (2), depends on the number of selected ground motion records, and is clearly equal to zero if the number of selected records is the same as the number of all the records from the given set of ground motion records. The ErrorTerm is presented in Figure 9 for both sets of ground motion records and for both optimization techniques. If the number of ground motion records is equal to or greater than 2, then the error is less than about 5% and decreases with the number of ground motion records.

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Free-field set of ground motion records			Near-field set of ground motion records		
	Ground motion ID			Ground motion ID	
Precedence list	GA	Simple	Precedence list	GA	Simple
1	22	22	1	51	51
2	7	7	2	30	49
3	17	21	3	40	40
4	6	6	4	28	28
5	21	17	5	49	45
6	10	10	6	35	43
7	18	18	7	43	25
8	16	4	8	45	30
9	23	9	9	25	27
10	19	5	10	36	36
11	13	13	11	37	37
12	4	19	12	50	50
13	24	23	13	31	31
14	8	8	14	33	33
15	9	2	15	53	38
16	15	16	16	48	42
17	2	14	17	32	35
18	20	20	18	27	46
19	14	12	19	38	39
20	5	3	20	42	32
21	12	11	21	39	41
22	3	15	22	26	48
23	11	24	23	34	34
24	1	1	24	52	52
			25	29	29
			26	44	26
			27	41	53
			28	47	47
			29	54	54
			30	46	44

Table II. The ground motion record precedence list for both sets of ground motion records, based on the GA and simple optimization techniques.

## 4.5. IDA analysis for the MDOF model

Steps 6 and 7 of the methodology (Section 2) are illustrated in this section, whereas step 8, the seismic performance assessment of the test structure, is outside the scope of this paper. Nevertheless, in addition to the top displacement, the results are presented also in terms of the maximum interstorey drift ratio, which is usually a sufficiently good parameter for defining different limit states (performance levels).

The single-record IDA curves for the MDOF model were calculated gradually, starting from the first ground motion record in the predefined precedence list (Table II). The 'tolerance' (Equation (4)) was calculated after each single-record IDA curve was determined (Figure 10). For clarity of exposition, the single-record IDA curves were calculated for all the ground motion records within two sets of ground motion records, although there was no need to do so, because the tolerance



Figure 10. The tolerance function *versus* the number of selected ground motion records for the MDOF model, for both optimization techniques and for two different damage measures: (left) based on the free-field ground motion records and (right) based on the near-field ground motion records.

for the selected IDA curve, based on a number of ground motion records equal to the first four or more, is very low and drops below the acceptable tolerance level of 10%. It has, therefore, been concluded that the first four ground motion records from the precedence list are sufficient to predict the median IDA curve.

In general, the tolerance function decreases if the number of selected ground motion records increases. For the first four and further selected ground motion records, the tolerance does not change significantly, especially for the near-field set of ground motion records, which once again confirms the previous observation.

The 'selected' median IDA curve, determined on the basis of an IDA analysis of only the first four ground motion records from the precedence list (Table II), is presented in Figures 11 and 12, in terms of the top displacement and maximum inter-storey drifts, respectively. From these figures it can be seen that there is very good agreement, for both sets of ground motion records, between the 'selected' median IDA curves, based on the two different procedures, and the 'original' median IDA curve.

# 4.6. Discussion of the results

In order to measure the error between the 'selected' median IDA curves and the 'original' median IDA curve, an ErrorTerm, as had been defined for the simple model (Equation (2)), was determined for the MDOF model. This error is presented in Figure 13 for different damage measures, for both of the optimization techniques employed to determine the precedence list and for both sets of ground motion records. It can be seen that the error (Figure 13) is less than 10% if four or more ground motion records are used to determine the 'selected' median IDA curve. Clearly, a larger error is observed for the maximum inter-storey drift ratio in comparison with the error determined for the top displacement. This conclusion was expected since the precedence list was determined for the SDOF model, which can predict only the top displacement. It is also clear that the error determined based on the SDOF model (Figure 9) is smaller than the error for the MDOF model.



Figure 11. The single-record IDA curves, the first four selected single-record IDA curves, the selected and the original median IDA curve for the MDOF model. The damage measure is the top displacement. The results are presented for the free- and near-field sets of ground motion records and for both optimization techniques (GA and simple procedure).

The difference arises from the fact that the error is minimized only for the SDOF model, and it is assumed that this error will also be small in the case of the MDOF model. The GA and the simple procedure for the determination of the precedence list can be used to minimize the error practically to the same extent.

In the example, it was proved that the SDOF model was a sufficient representative of the simple model. Although the force–displacement relationship of the SDOF model is determined from the pushover curve, it was shown, based on a small parametric study, that even rough judgement for the force–displacement relationship of the SDOF model may be satisfactory. However, in this case, usually more than four ground motion records are needed for good prediction of the median IDA curve. For different structures, especially for buildings or bridges, which are not first mode dominant, the SDOF model may not be a sufficient representative of the simple model. Additional



Figure 12. The single-record IDA curves, the first four selected single-record IDA curves, the selected and the original median IDA curve for the MDOF model. The damage measure is the maximum inter-storey drift. The results are presented for the free- and near-field sets of ground motion records and for both optimization techniques (GA and simple procedure).

studies are, therefore, needed in order to define sufficient simple models for different types of structures.

Although this study is focused only on the predicting of the median IDA curve, the 16 and 84% fractile curves can be determined based on the IDA analysis performed on the SDOF model, similarly as has been shown in the case of other simplified methods for the determination of approximate median curves (e.g. [2, 3]).

The proposed procedure for the prediction of the median IDA curve is, see Figure 9, more efficient if the parity of the total number of ground motion records within one set of ground motion records (n), as well as the number of selected ground motion records (s), matches. For example, from Figure 9 it can be observed that the error is smaller if s is an even number since for this example n is also an even number.



Figure 13. The ErrorTerm *versus* the number of selected ground motion records for the MDOF model and for both optimization techniques: (left) based on the free-field ground motion records and (right) based on near-field ground motion records.

#### 5. CONCLUSIONS

A methodology has been proposed in order to predict the median IDA curve with only a limited number of ground motion records from a given set of records. For this purpose, the concept of a precedence list of ground motion records has been introduced. Determination of the precedence list of ground motion records is an optimization problem, which is solved in the paper by two different optimization techniques. In the proposed methodology, as in many other simplified methods, the response of a simple (e.g. SDOF) model is taken into account. Such an approach is not computationally demanding, and can substantially decrease the number of nonlinear dynamic analyses needed for sufficiently accurate prediction of the median IDA curve.

The methodology was applied to a three-storey-reinforced concrete frame building, using two sets of ground motion records, one a free-field set and the other a near-field set. It was proved that, for this particular example, the median IDA curve can be predicted with acceptable accuracy by employing only four ground motion records instead of 24 or 30, which is the number of all ground motion records in the free- and near-field sets of records, respectively. The error in the prediction of the median IDA curve, in terms of the top displacement or maximum inter-storey drift, is less then 10%. The genetic algorithm and the proposed simple procedure are efficient in solving the optimization problem of the ground motion precedence list.

The applicability of the methodology is limited at this stage of the study, since the method was validated only for a typical first mode dominant structure for which it was proven that the SDOF model is adequate to predict the precedence list. Obviously, the sufficiency of the SDOF model and the applicability of other possible simple models, which can be used for determination of precedence list, are still open to be investigated especially for structures with significant higher mode effects.

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